GATE SOLVED PAPER
Electronics & Communication
Signals & Systems

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GATE SOLVED PAPER - EC

SIGNALS & SYSTEMS

2013 **ONE MARK**

Q. 1 Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by

(A) product of $h_1(t)$ and $h_2(t)$

(B) sum of $h_1(t)$ and $h_2(t)$

(C) convolution of $h_1(t)$ and $h_2(t)$

(D) subtraction of $h_2(t)$ from $h_1(t)$

Q. 2 The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is

(A) $\frac{t^2}{2}u(t)$

(B) $\frac{t(t-1)}{2}u(t-1)$

(C) $\frac{(t-1)^2}{2}u(t-1)$

(D) $\frac{t^2-1}{2}u(t-1)$

Q. 3 For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$, the fundamental frequency in rad/s

(A) 100

(B) 300

(C) 500

(D) 1500

Q. 4 A band-limited signal with a maximum frequency of 5 kHz is to be sampled. According to the sampling theorem, the sampling frequency which is not valid is

(A) 5 kHz

(B) 12 kHz

(C) 15 kHz

(D) 20 kHz

Q. 5 Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?

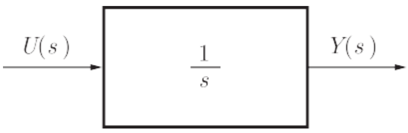
(A) All the poles of the system must lie on the left side of the $j\omega$ axis

(B) Zeros of the system can lie anywhere in the s-plane

(C) All the poles must lie within $|s| = 1$

(D) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

Q. 6 Assuming zero initial condition, the response $y(t)$ of the system given below to a unit step input $u(t)$ is



(A) $u(t)$

(B) $tu(t)$

(C) $\frac{t^2}{2}u(t)$

(D) $e^{-t}u(t)$

Q. 7 Let $g(t) = e^{-\pi t^2}$, and $h(t)$ is a filter matched to $g(t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is

(A) $e^{-\pi f^2}$ (B) $e^{-\pi f^2/2}$

(C) $e^{-\pi |f|}$ (D) $e^{-2\pi f^2}$

2013 TWO MARKS

Q. 8 The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is

(A) 0 (B) 1

(C) 2 (D) 3

Q. 9 A system described by the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = x(t)$. Let $x(t)$ be a rectangular pulse given by

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & otherwise \end{cases}$$

Assuming that $y(0) = 0$ and $\frac{dy}{dt} = 0$ at $t = 0$, the Laplace transform of $y(t)$ is

(A) $\frac{e^{-2s}}{s(s+2)(s+3)}$ (B) $\frac{1-e^{-2s}}{s(s+2)(s+3)}$

(C) $\frac{e^{-2s}}{(s+2)(s+3)}$ (D) $\frac{1-e^{-2s}}{(s+2)(s+3)}$

Q. 10 A system described by a linear, constant coefficient, ordinary, first order differential equation has an exact solution given by $y(t)$ for $t > 0$, when the forcing function is $x(t)$ and the initial condition is $y(0)$. If one wishes to modify the system so that the solution becomes $-2y(t)$ for $t > 0$, we need to

(A) change the initial condition to $-y(0)$ and the forcing function to $2x(t)$

(B) change the initial condition to $2y(0)$ and the forcing function to $-x(t)$

(C) change the initial condition to $j\sqrt{2}\,y(0)$ and the forcing function to $j\sqrt{2}\,x(t)$

(D) change the initial condition to $-2y(0)$ and the forcing function to $-2x(t)$

Q. 11 The DFT of a vector $[a\ b\ c\ d]$ is the vector $[\alpha\ \beta\ \gamma\ \delta]$. Consider the product

$$[p\ q\ r\ s] = [a\ b\ c\ d]\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of the vector $[p\ q\ r\ s]$ is a scaled version of

(A) $[\alpha^2\ \beta^2\ \gamma^2\ \delta^2]$ (B) $[\sqrt{\alpha}\ \sqrt{\beta}\ \sqrt{\gamma}\ \sqrt{\delta}]$

(C) $[\alpha+\beta\ \ \beta+\delta\ \ \delta+\gamma\ \ \gamma+\alpha]$ (D) $[\alpha\ \beta\ \gamma\ \delta]$

2012 ONE MARK

Q. 12 The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $tf(t)$ is

(A) $-\frac{s}{(s^2+s+1)^2}$ (B) $-\frac{2s+1}{(s^2+s+1)^2}$

(C) $\frac{s}{(s^2+s+1)^2}$ (D) $\frac{2s+1}{(s^2+s+1)^2}$

Q. 13 If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its z-transform in the z-plane will be

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{1}{3} < |z| < \frac{1}{2}$

(C) $\frac{1}{2} < |z| < 3$ (D) $\frac{1}{3} < |z|$

2012 TWO MARKS

Q. 14 The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is

(A) time-invariant and stable (B) stable and not time-invariant

(C) time-invariant and not stable (D) not time-invariant and not stable

Q. 15 The Fourier transform of a signal $h(t)$ is $H(jw) = (2\cos w)(\sin 2w)/w$. The value of $h(0)$ is

(A) 1/4 (B) 1/2

(C) 1 (D) 2

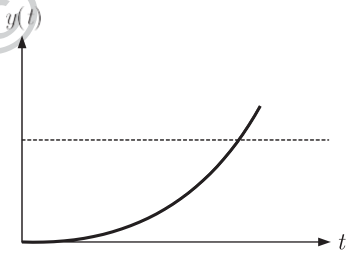
Q. 16 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals

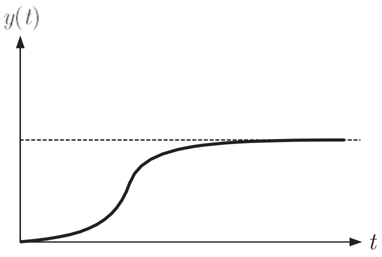
(A) 0 (B) 1/2

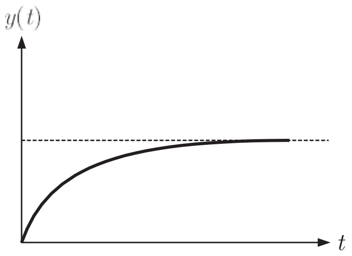
(C) 1 (D) 3/2

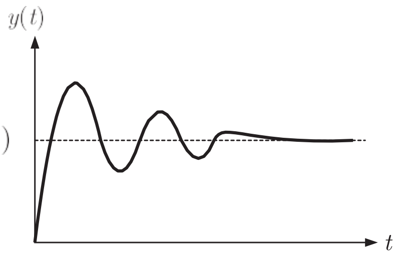
2011 ONE MARK

Q. 17 The differential equation $100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$ describes a system with an input $x(t)$ and an output $y(t)$. The system, which is initially relaxed, is excited by a unit step input. The output $y(t)$ can be represented by the waveform

(A)


(B)


(C)


(D)


Q. 18 The trigonometric Fourier series of an even function does not have the

(A) dc term
(B) cosine terms
(C) sine terms
(D) odd harmonic terms

Q. 19 A system is defined by its impulse response $h(n) = 2^n u(n-2)$. The system is

(A) stable and causal
(B) causal but not stable
(C) stable but not causal
(D) unstable and non-causal

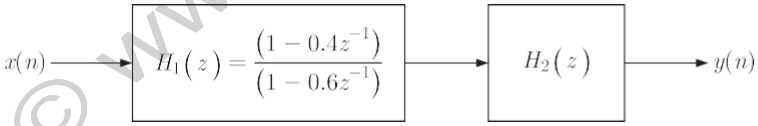
Q. 20 If the unit step response of a network is $(1 - e^{-\alpha t})$, then its unit impulse response is

(A) $\alpha e^{-\alpha t}$
(B) $\alpha^{-1} e^{-\alpha t}$
(C) $(1 - \alpha^{-1}) e^{-\alpha t}$
(D) $(1 - \alpha) e^{-\alpha t}$

2011 **TWO MARKS**

Q. 21 An input $x(t) = \exp(-2t)\, u(t) + \delta(t-6)$ is applied to an LTI system with impulse response $h(t) = u(t)$. The output is (A)
$[1 - \exp(-2t)]\, u(t) + u(t+6)$
(B) $[1 - \exp(-2t)]\, u(t) + u(t-6)$
(C) $0.5[1 - \exp(-2t)]\, u(t) + u(t+6)$
(D) $0.5[1 - \exp(-2t)]\, u(t) + u(t-6)$

Q. 22 Two systems $H_1(Z)$ and $H_2(Z)$ are connected in cascade as shown below. The overall output $y(n)$ is the same as the input $x(n)$ with a one unit delay. The transfer function of the second system $H_2(Z)$ is



(A) $\dfrac{1 - 0.6z^{-1}}{z^{-1}(1 - 0.4z^{-1})}$
(B) $\dfrac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$
(C) $\dfrac{z^{-1}(1 - 0.4z^{-1})}{(1 - 0.6z^{-1})}$
(D) $\dfrac{1 - 0.4z^{-1}}{z^{-1}(1 - 0.6z^{-1})}$

Q. 23 The first six points of the 8-point DFT of a real valued sequence are $5,\ 1 - j3,\ 0,\ 3 - j4,\ 0$ and $3 + j4$. The last two points of the DFT are respectively

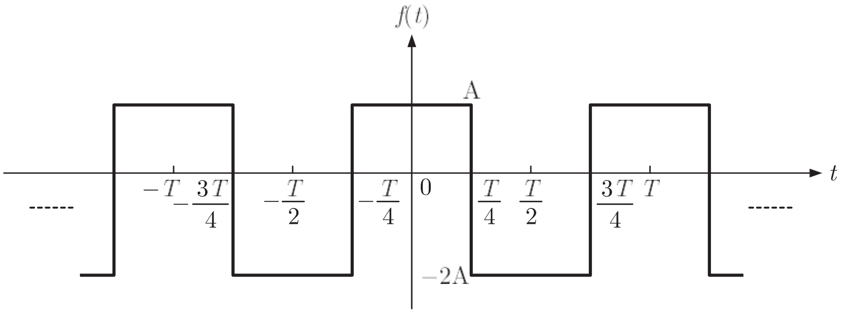
(A) $0,\ 1 - j3$
(B) $0,\ 1 + j3$
(C) $1 + j3,\ 5$
(D) $1 - j3,\ 5$

2010 **ONE MARK**

Q. 24 Consider the z-transform $x(z) = 5z^2 + 4z^{-1} + 3;\ 0 < |z| < 3$. The inverse z-transform $x[n]$ is

(A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$
(B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$
(C) $5u[n+2] + 3u[n] + 4u[n-1]$
(D) $5u[n-2] + 3u[n] + 4u[n+1]$

Q. 25 The trigonometric Fourier series for the waveform $f(t)$ shown below contains



(A) only cosine terms and zero values for the dc components

(B) only cosine terms and a positive value for the dc components

(C) only cosine terms and a negative value for the dc components

(D) only sine terms and a negative value for the dc components

Q. 26 Two discrete time system with impulse response $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

(A) $\delta[n-1] + \delta[n-2]$ (B) $\delta[n-4]$

(C) $\delta[n-3]$ (D) $\delta[n-1]\,\delta[n-2]$

Q. 27 For a N-point FET algorithm $N = 2^m$ which one of the following statements is TRUE ?

(A) It is not possible to construct a signal flow graph with both input and output in normal order

(B) The number of butterflies in the m^{th} stage in N / m

(C) In-place computation requires storage of only 2N data

(D) Computation of a butterfly requires only one complex multiplication.

2010 TWO MARKS

Q. 28 Given $f(t) = L^{-1}\left[\frac{3s+1}{s^3+4s^2+(k-3)s}\right]$. If $\lim_{t \to \infty} f(t) = 1$, then the value of k is

(A) 1 (B) 2

(C) 3 (D) 4

Q. 29 A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Assuming zero initial conditions, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by

(A) $(e^{t} - e^{3t})u(t)$ (B) $(e^{-t} - e^{-3t})u(t)$

(C) $(e^{-t} + e^{-3t})u(t)$ (D) $(e^{t} + e^{3t})u(t)$

Q. 30 The transfer function of a discrete time LTI system is given by

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Consider the following statements:

S1: The system is stable and causal for ROC: $|z| > 1/2$

S2: The system is stable but not causal for ROC: $|z| < 1/4$

S3: The system is neither stable nor causal for ROC: $1/4 < |z| < 1/2$

Which one of the following statements is valid ?

(A) Both S1 and S2 are true (B) Both S2 and S3 are true

(C) Both S1 and S3 are true (D) S1, S2 and S3 are all true

2009 ONE MARK

Q. 31 The Fourier series of a real periodic function has only

(P) cosine terms if it is even

(Q) sine terms if it is even

(R) cosine terms if it is odd

(S) sine terms if it is odd

Which of the above statements are correct ?

(A) P and S (B) P and R

(C) Q and S (D) Q and R

Q. 32 A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is true ?

(A) f has frequency components at 0 and $\frac{1}{2\pi}$ Hz

(B) f has frequency components at 0 and $\frac{1}{\pi}$ Hz

(C) f has frequency components at $\frac{1}{2\pi}$ and $\frac{1}{\pi}$ Hz

(D) f has frequency components at $\frac{0.1}{2\pi}$ and $\frac{1}{\pi}$ Hz

Q. 33 The ROC of z-transform of the discrete time sequence

$$x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1) \text{ is}$$

(A) $\frac{1}{3} < |z| < \frac{1}{2}$ (B) $|z| > \frac{1}{2}$

(C) $|z| < \frac{1}{3}$ (D) $2 < |z| < 3$

2009 TWO MARKS

Q. 34 Given that $F(s)$ is the one-side Laplace transform of $f(t)$, the Laplace transform of $\int_0^t f(\tau)\,d\tau$ is

(A) $sF(s) - f(0)$ (B) $\frac{1}{s}F(s)$

(C) $\int_0^s F(\tau)\,d\tau$ (D) $\frac{1}{s}[F(s) - f(0)]$

Q. 35 A system with transfer function $H(z)$ has impulse response $h(.)$ defined as $h(2) = 1$, $h(3) = -1$ and $h(k) = 0$ otherwise. Consider the following statements.

S1 : $H(z)$ is a low-pass filter.

S2 : $H(z)$ is an FIR filter.

Which of the following is correct?

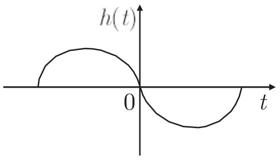
(A) Only S2 is true

(B) Both S1 and S2 are false

(C) Both S1 and S2 are true, and S2 is a reason for S1

(D) Both S1 and S2 are true, but S2 is not a reason for S1

Q. 36 Consider a system whose input x and output y are related by the equation
$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, g(2\tau)\, d\tau$ where $h(t)$ is shown in the graph.



Which of the following four properties are possessed by the system ?
BIBO : Bounded input gives a bounded output.
Causal : The system is causal,
LP : The system is low pass.
LTI : The system is linear and time-invariant.

(A) Causal, LP
(B) BIBO, LTI
(C) BIBO, Causal, LTI
(D) LP, LTI

Q. 37 The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence {1,0,2,3} is

(A) $[0, -2+2j, 2, -2-2j]$
(B) $[2, 2+2j, 6, 2-2j]$
(C) $[6, 1-3j, 2, 1+3j]$
(D) $[6, -1+3j, 0, -1-3j]$

Q. 38 An LTI system having transfer function $\frac{s^2+1}{s^2+2s+1}$ and input $x(t) = \sin(t+1)$ is in steady state. The output is sampled at a rate w_s rad / s to obtain the final output $\{x(k)\}$. Which of the following is true ?

(A) $y(.)$ is zero for all sampling frequencies w_s
(B) $y(.)$ is nonzero for all sampling frequencies w_s
(C) $y(.)$ is nonzero for $w_s > 2$, but zero for $w_s < 2$
(D) $y(.)$ is zero for $w_s > 2$, but nonzero for $w_2 < 2$

2008 ONE MARK

Q. 39 The input and output of a continuous time system are respectively denoted by $x(t)$ and $y(t)$. Which of the following descriptions corresponds to a causal system ?

(A) $y(t) = x(t-2) + x(t+4)$
(B) $y(t) = (t-4)x(t+1)$
(C) $y(t) = (t+4)x(t-1)$
(D) $y(t) = (t+5)x(t+5)$

Q. 40 The impulse response $h(t)$ of a linear time invariant continuous time system is described by $h(t) = \exp(at)u(t) + \exp(bt)u(-t)$ where $u(-t)$ denotes the unit step function, and a and b are real constants. This system is stable if

(A) a is positive and b is positive
(B) a is negative and b is negative
(C) a is negative and b is negative
(D) a is negative and b is positive

2008 TWO MARKS

Q. 41 A linear, time - invariant, causal continuous time system has a rational transfer function with simple poles at $s = -2$ and $s = -4$ and one simple zero at $s = -1$.

A unit step $u(t)$ is applied at the input of the system. At steady state, the output has constant value of 1. The impulse response of this system is

(A) $[\exp(-2t) + \exp(-4t)]\,u(t)$

(B) $[-4\exp(-2t) - 12\exp(-4t) - \exp(-t)]\,u(t)$

(C) $[-4\exp(-2t) + 12\exp(-4t)]\,u(t)$

(D) $[-0.5\exp(-2t) + 1.5\exp(-4t)]\,u(t)$

Q. 42 The signal $x(t)$ is described by

$$x(t) = \begin{cases} 1 & \text{for } -1 \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$$

Two of the angular frequencies at which its Fourier transform becomes zero are

(A) π, 2π (B) 0.5π, 1.5π

(C) 0, π (D) 2π, 2.5π

Q. 43 A discrete time linear shift - invariant system has an impulse response $h[n]$ with $h[0] = 1$, $h[1] = -1$, $h[2] = 2$, and zero otherwise The system is given an input sequence $x[n]$ with $x[0] = x[2] = 1$, and zero otherwise. The number of nonzero samples in the output sequence $y[n]$, and the value of $y[2]$ are respectively

(A) 5, 2 (B) 6, 2

(C) 6, 1 (D) 5, 3

Q. 44 Let $x(t)$ be the input and $y(t)$ be the output of a continuous time system. Match the system properties P1, P2 and P3 with system relations R1, R2, R3, R4

Properties	Relations
P1 : Linear but NOT time - invariant	R1 : $y(t) = t^2 x(t)$
P2 : Time - invariant but NOT linear	R2 : $y(t) = t\lvert x(t)\rvert$
P3 : Linear and time - invariant	R3 : $y(t) = \lvert x(t)\rvert$
	R4 : $y(t) = x(t-5)$

(A) (P1, R1), (P2, R3), (P3, R4) (B) (P1, R2), (P2, R3), (P3, R4)

(C) (P1, R3), (P2, R1), (P3, R2) (D) (P1, R1), (P2, R2), (P3, R3)

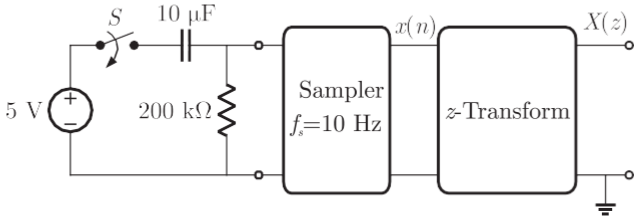
Q. 45 $\{x(n)\}$ is a real - valued periodic sequence with a period N. $x(n)$ and $X(k)$ form N-point Discrete Fourier Transform (DFT) pairs. The DFT $Y(k)$ of the sequence $y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\,x(n+r)$ is

(A) $\lvert X(k)\rvert^2$ (B) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)\,X(k+r)$

(C) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)\,X(k+r)$ (D) 0

Statement for Linked Answer Question 46 and 47:

In the following network, the switch is closed at $t = 0^-$ and the sampling starts from $t = 0$. The sampling frequency is 10 Hz.



Q. 46 The samples $x(n),\ n = (0, 1, 2, ...)$ are given by

(A) $5(1 - e^{-0.05n})$ (B) $5e^{-0.05n}$

(C) $5(1 - e^{-5n})$ (D) $5e^{-5n}$

Q. 47 The expression and the region of convergence of the z -transform of the sampled signal are

(A) $\frac{5z}{z - e^{5}},\ |z| < e^{-5}$ (B) $\frac{5z}{z - e^{-0.05}},\ |z| < e^{-0.05}$

(C) $\frac{5z}{z - e^{-0.05}},\ |z| > e^{-0.05}$ (D) $\frac{5z}{z - e^{-5}},\ |z| > e^{-5}$

Statement for Linked Answer Question 48 & 49:

The impulse response $h(t)$ of linear time - invariant continuous time system is given by $h(t) = \exp(-2t)\,u(t)$, where $u(t)$ denotes the unit step function.

Q. 48 The frequency response $H(w)$ of this system in terms of angular frequency w, is given by $H(w)$

(A) $\frac{1}{1 + j2w}$ (B) $\frac{\sin w}{w}$

(C) $\frac{1}{2 + jw}$ (D) $\frac{jw}{2 + jw}$

Q. 49 The output of this system, to the sinusoidal input $x(t) = 2\cos 2t$ for all time t, is

(A) 0 (B) $2^{-0.25}\cos(2t - 0.125p)$

(C) $2^{-0.5}\cos(2t - 0.125p)$ (D) $2^{-0.5}\cos(2t - 0.25p)$

2007 **ONE MARK**

Q. 50 If the Laplace transform of a signal $Y(s) = \frac{1}{s(s-1)}$, then its final value is

(A) -1 (B) 0

(C) 1 (D) Unbounded

2007 **TWO MARKS**

Q. 51 The 3-dB bandwidth of the low-pass signal $e^{-t}u(t)$, where $u(t)$ is the unit step function, is given by

(A) $\frac{1}{2p}$ Hz (B) $\frac{1}{2p}\sqrt{\sqrt{2} - 1}$ Hz

(C) 3 (D) 1 Hz

Q. 52 A 5-point sequence $x[n]$ is given as $x[-3] = 1,\ x[-2] = 1,\ x[-1] = 0,\ x[0] = 5$ and $x[1] = 1$. Let $X(e^{iw})$ denoted the discrete-time Fourier transform of $x[n]$. The value of $\int_{-p}^{p} X(e^{jw})\,dw$ is

(A) 5 (B) $10p$

(C) $16p$ (D) $5 + j10p$

Q. 53 The z -transform $X(z)$ of a sequence $x[n]$ is given by $X[z] = \frac{0.5}{1}$. It is given that the region of convergence of $X(z)$ includes the unit circle. The value of $x[0]$ is

(A) -0.5 (B) 0

(C) 0.25 (D) 05

Q. 54 A Hilbert transformer is a

(A) non-linear system
(B) non-causal system
(C) time-varying system
(D) low-pass system

Q. 55 The frequency response of a linear, time-invariant system is given by $H(f) = \frac{5}{1+j10\pi f}$. The step response of the system is

(A) $5(1-e^{-5t})u(t)$
(B) $5\left(1-e^{-\frac{t}{5}}\right)u(t)$
(C) $\frac{1}{2}(1-e^{-5t})u(t)$
(D) $\frac{1}{5}\left(1-e^{-\frac{t}{5}}\right)u(t)$

2006 ONE MARK

Q. 56 Let $x(t) \leftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal $x(5t-3)$ in terms of $X(j\omega)$ is given as

(A) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$
(B) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$
(C) $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$
(D) $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

Q. 57 The Dirac delta function $\delta(t)$ is defined as

(A) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(B) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

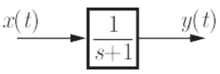
(C) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)\,dt = 1$

(D) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty}\delta(t)\,dt = 1$

Q. 58 If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$ then the region of convergence of $x_1[n] - x_2[n]$ includes

(A) $\frac{1}{3} < |z| < 3$
(B) $\frac{2}{3} < |z| < 3$
(C) $\frac{3}{2} < |z| < 3$
(D) $\frac{1}{3} < |z| < \frac{2}{3}$

Q. 59 In the system shown below, $x(t) = (\sin t)\,u(t)$ In steady-state, the response $y(t)$ will be



(A) $\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$
(B) $\frac{1}{\sqrt{2}}\sin\left(t + \frac{\pi}{4}\right)$
(C) $\frac{1}{\sqrt{2}}e^{-t}\sin t$
(D) $\sin t - \cos t$

2006 TWO MARKS

Q. 60 Consider the function $f(t)$ having Laplace transform

$$F(s) = \frac{\omega_0}{s^2 + \omega_0^2} \quad \text{Re}[s] > 0$$

The final value of $f(t)$ would be

(A) 0 (B) 1

(C) $-1 \le f(\infty) \le 1$ (D) ∞

Q. 61 A system with input $x[n]$ and output $y[n]$ is given as $y[n] = (\sin \frac{5}{6}\pi n)\, x[n]$. The system is

(A) linear, stable and invertible

(B) non-linear, stable and non-invertible

(C) linear, stable and non-invertible

(D) linear, unstable and invertible

Q. 62 The unit step response of a system starting from rest is given by $c(t) = 1 - e^{-2t}$ for $t \ge 0$. The transfer function of the system is

(A) $\frac{1}{1+2s}$ (B) $\frac{2}{2+s}$

(C) $\frac{1}{2+s}$ (D) $\frac{2s}{1+2s}$

Q. 63 The unit impulse response of a system is $f(t) = e^{-t}, t \ge 0$. For this system the steady-state value of the output for unit step input is equal to

(A) -1 (B) 0

(C) 1 (D) ∞

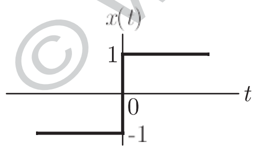
2005 **ONE MARK**

Q. 64 Choose the function $f(t); -\infty < t < \infty$ for which a Fourier series cannot be defined.

(A) $3\sin(25t)$ (B) $4\cos(20t+3) + 2\sin(710t)$

(C) $\exp(-|t|)\sin(25t)$ (D) 1

Q. 65 The function $x(t)$ is shown in the figure. Even and odd parts of a unit step function $u(t)$ are respectively,



(A) $\frac{1}{2}, \frac{1}{2}x(t)$ (B) $-\frac{1}{2}, \frac{1}{2}x(t)$

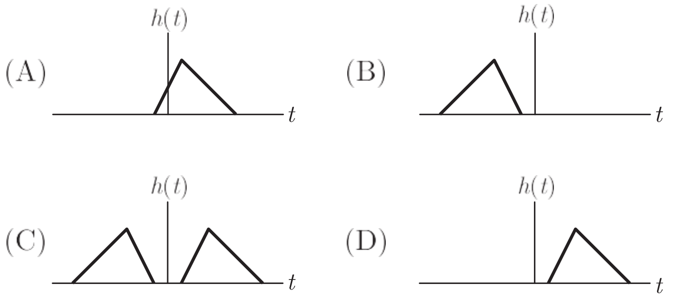
(C) $\frac{1}{2}, -\frac{1}{2}x(t)$ (D) $-\frac{1}{2}, -\frac{1}{2}x(t)$

Q. 66 The region of convergence of $z-$ transform of the sequence $\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be

(A) $|z| < \frac{5}{6}$ (B) $|z| > \frac{5}{6}$

(C) $\frac{5}{6} < |z| < \frac{6}{5}$ (D) $\frac{6}{5} < |z| < \infty$

Q. 67 Which of the following can be impulse response of a causal system ?



Q. 68 Let $x(n) = (\frac{1}{2})^n u(n), y(n) = x^2(n)$ and $Y(e^{j\omega})$ be the Fourier transform of $y(n)$ then $Y(e^{j0})$

(A) $\frac{1}{4}$

(B) 2

(C) 4

(D) $\frac{4}{3}$

Q. 69 The power in the signal $s(t) = 8\cos(20\pi - \frac{\pi}{2}) + 4\sin(15\pi t)$ is

(A) 40

(B) 41

(C) 42

(D) 82

2005 **TWO MARKS**

Q. 70 The output $y(t)$ of a linear time invariant system is related to its input $x(t)$ by the following equations

$$y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d + T)$$

The filter transfer function $H(\omega)$ of such a system is given by

(A) $(1 + \cos\omega T)e^{-j\omega t_d}$

(B) $(1 + 0.5\cos\omega T)e^{-j\omega t_d}$

(C) $(1 - \cos\omega T)e^{-j\omega t_d}$

(D) $(1 - 0.5\cos\omega T)e^{-j\omega t_d}$

Q. 71 Match the following and choose the correct combination.

Group 1

E. Continuous and aperiodic signal

F. Continuous and periodic signal

G. Discrete and aperiodic signal

H. Discrete and periodic signal

Group 2

1. Fourier representation is continuous and aperiodic
2. Fourier representation is discrete and aperiodic
3. Fourier representation is continuous and periodic
4. Fourier representation is discrete and periodic

(A) E − 3, F − 2, G − 4, H − 1

(B) E − 1, F − 3, G − 2, H − 4

(C) E − 1, F − 2, G − 3, H − 4

(D) E − 2, F − 1, G − 4, H − 3

Q. 72 A signal $x(n) = \sin(w_0 n + f)$ is the input to a linear time- invariant system having a frequency response $H(e^{jw})$. If the output of the system $Ax(n - n_0)$ then the most general form of $+H(e^{jw})$ will be

(A) $-n_0 w_0 + b$ for any arbitrary real

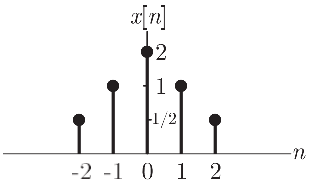
(B) $-n_0 w_0 + 2pk$ for any arbitrary integer k

(C) $n_0 w_0 + 2pk$ for any arbitrary integer k

(D) $-n_0 w_0 f$

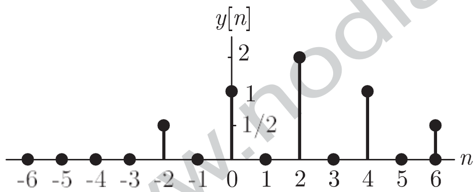
Statement of linked answer question 73 and 74 :

A sequence $x(n)$ has non-zero values as shown in the figure.

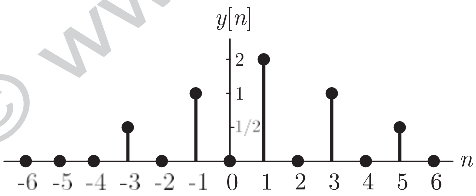


Q. 73 The sequence $y(n) = \begin{cases} x\left(\frac{n}{2} - 1\right), & \text{For } n \text{ even} \\ 0, & \text{For } n \text{ odd} \end{cases}$ will be

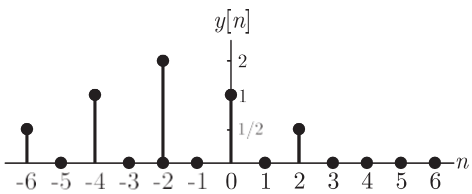
(A)



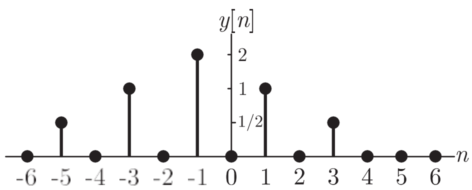
(B)



(C)



(D)



Q. 74 The Fourier transform of $y(2n)$ will be

(A) $e^{-j2\omega}[\cos 4\omega + 2\cos 2\omega + 2]$ (B) $\cos 2\omega + 2\cos \omega + 2$

(C) $e^{-j\omega}[\cos 2\omega + 2\cos \omega + 2]$ (D) $e^{-j2\omega}[\cos 2\omega + 2\cos + 2]$

Q. 75 For a signal $x(t)$ the Fourier transform is $X(f)$. Then the inverse Fourier transform of $X(3f + 2)$ is given by

(A) $\frac{1}{2}x\left(\frac{t}{2}\right)e^{j3\pi t}$ (B) $\frac{1}{3}x\left(\frac{t}{3}\right)e^{-\frac{j4\pi t}{3}}$

(C) $3x(3t)e^{-j4\pi t}$ (D) $x(3t + 2)$

2004 ONE MARK

Q. 76 The impulse response $h[n]$ of a linear time-invariant system is given by $h[n] = u[n+3] + u[n-2] - 2n[n-7]$ where $u[n]$ is the unit step sequence. The above system is

(A) stable but not causal (B) stable and causal

(C) causal but unstable (D) unstable and not causal

Q. 77 The z-transform of a system is $H(z) = \frac{z}{z - 0.2}$. If the ROC is $|z| < 0.2$, then the impulse response of the system is

(A) $(0.2)^n u[n]$ (B) $(0.2)^n u[-n-1]$

(C) $-(0.2)^n u[n]$ (D) $-(0.2)^n u[-n-1]$

Q. 78 The Fourier transform of a conjugate symmetric function is always

(A) imaginary (B) conjugate anti-symmetric

(C) real (D) conjugate symmetric

2004 TWO MARKS

Q. 79 Consider the sequence $x[n] = [-4 - j51 + j25]$. The conjugate anti-symmetric part of the sequence is

(A) $[-4 - j2.5,\ j2,\ 4 - j2.5]$ (B) $[-j2.5,\ 1,\ j2.5]$

(C) $[-j2.5,\ j2,\ 0]$ (D) $[-4,\ 1,\ 4]$

Q. 80 A causal LTI system is described by the difference equation

$$2y[n] = ay[n-2] - 2x[n] + bx[n-1]$$

The system is stable only if

(A) $|a| = 2,\ |b| < 2$ (B) $|a| > 2,\ |b| > 2$

(C) $|a| < 2$, any value of b (D) $|b| < 2$, any value of a

Q. 81 The impulse response $h[n]$ of a linear time invariant system is given as

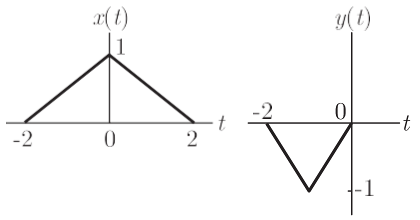
$$h[n] = \begin{cases} -2\sqrt{2} & n = 1, -1 \\ 4\sqrt{2} & n = 2, -2 \\ 0 & \text{otherwise} \end{cases}$$

If the input to the above system is the sequence $e^{j\pi n/4}$, then the output is

(A) $4\sqrt{2}\,e^{j\pi n/4}$ (B) $4\sqrt{2}\,e^{-j\pi n/4}$

(C) $4e^{j\pi n/4}$ (D) $-4e^{j\pi n/4}$

Q. 82 Let $x(t)$ and $y(t)$ with Fourier transforms $F(f)$ and $Y(f)$ respectively be related as shown in Fig. Then $Y(f)$ is



(A) $-\frac{1}{2}X(f/2)e^{-jpf}$ (B) $-\frac{1}{2}X(f/2)e^{j2pf}$

(C) $-X(f/2)e^{j2pf}$ (D) $-X(f/2)e^{-j2pf}$

2003 ONE MARK

Q. 83 The Laplace transform of $i(t)$ is given by $I(s) = \frac{2}{s(1+s)}$ At t " 3, The value of $i(t)$ tends to

(A) 0

(B) 1

(C) 2

(D) 3

Q. 84 The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by $g_p(t) = \sum_{n=-3} c_n e^{j2pf_0 t}$. It is given that $c_3 = 3 + j5$. Then c_{-3} is

(A) $5 + j3$ (B) $-3 - j5$

(C) $-5 + j3$ (D) $3 - j5$

Q. 85 Let $x(t)$ be the input to a linear, time-invariant system. The required output is $4p(t - 2)$. The transfer function of the system should be

(A) $4e^{j4pf}$ (B) $2e^{-j8pf}$

(C) $4e^{-j4pf}$ (D) $2e^{j8pf}$

Q. 86 A sequence $x(n)$ with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2d(n-3)$ where

$$d(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The output at $n = 4$ is

(A) -6 (B) zero

(C) 2 (D) -4

2003 TWO MARKS

Q. 87 Let P be linearity, Q be time-invariance, R be causality and S be stability. A discrete time system has the input-output relationship,

$$y(n) = \begin{cases} x(n) & n \$ 1 \\ 0, & n = 0 \\ x(n+1) & n \# -1 \end{cases}$$

where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties

(A) P, S but not Q, R
(B) P, Q, S but not R
(C) P, Q, R, S
(D) Q, R, S but not P

Common Data For Q. 88 & 89 :

The system under consideration is an RC low-pass filter (RC-LPF) with $R = 1$ k W and $C = 1.0\ m$ F.

Q. 88 Let $H(f)$ denote the frequency response of the RC-LPF. Let f_1 be the highest frequency such that $0 \# |f| \# f_1 \frac{|H(f_1)|}{H(0)} \$ 0.95$. Then f_1 (in Hz) is

(A) 324.8
(B) 163.9
(C) 52.2
(D) 104.4

Q. 89 Let $t_g(f)$ be the group delay function of the given RC-LPF and $f_2 = 100$ Hz. Then $t_g(f_2)$ in ms, is

(A) 0.717
(B) 7.17
(C) 71.7
(D) 4.505

2002 ONE MARK

Q. 90 Convolution of $x(t+5)$ with impulse function $d(t-7)$ is equal to

(A) $x(t-12)$
(B) $x(t+12)$
(C) $x(t-2)$
(D) $x(t+2)$

Q. 91 Which of the following cannot be the Fourier series expansion of a periodic signal?

(A) $x(t) = 2\cos t + 3\cos 3t$
(B) $x(t) = 2\cos pt + 7\cos t$
(C) $x(t) = \cos t + 0.5$
(D) $x(t) = 2\cos 1.5pt + \sin 3.5pt$

Q. 92 The Fourier transform $F\{e^{-1}u(t)\}$ is equal to $\frac{1}{1+j2pf}$. Therefore, $F\left\{\frac{1}{1+j2pt}\right\}$ is

(A) $e^{f}u(f)$
(B) $e^{-f}u(f)$
(C) $e^{f}u(-f)$
(D) $e^{-f}u(-f)$

Q. 93 A linear phase channel with phase delay T_p and group delay T_g must have

(A) $T_p = T_g =$ constant
(B) $T_p \setminus f$ and $T_g \setminus f$
(C) $T_p =$ constant and $T_g \setminus f$ (f denote frequency)
(D) $T_p \setminus f$ and $T_p =$ constant

2002 TWO MARKS

Q. 94 The Laplace transform of continuous - time signal $x(t)$ is $X(s) = \frac{5-s}{s^2-s-2}$. If the Fourier transform of this signal exists, the $x(t)$ is

(A) $e^{2t}u(t) - 2e^{-t}u(t)$
(B) $-e^{2t}u(-t) + 2e^{-t}u(t)$
(C) $-e^{2t}u(-t) - 2e^{-t}u(t)$
(D) $e^{2t}u(-t) - 2e^{-t}u(t)$

Q. 95 If the impulse response of discrete - time system is $h[n] = -5^n u[-n-1]$, then the system function $H(z)$ is equal to

(A) $\frac{-z}{z-5}$ and the system is stable
(B) $\frac{z}{z-5}$ and the system is stable
(C) $\frac{-z}{z-5}$ and the system is unstable
(D) $\frac{z}{z-5}$ and the system is unstable

2001 ONE MARK

Q. 96 The transfer function of a system is given by $H(s) = \frac{1}{s^2(s-2)}$. The impulse response of the system is

(A) $(t^2 * e^{-2t})u(t)$
(B) $(t * e^{2t})u(t)$
(C) $(te^{-2}t)u(t)$
(D) $(te^{-2t})u(t)$

Q. 97 The region of convergence of the $z-$ transform of a unit step function is

(A) $|z| > 1$
(B) $|z| < 1$
(C) (Real part of z) > 0
(D) (Real part of z) < 0

Q. 98 Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(A) 1
(B) -1
(C) 0
(D) $\frac{\pi}{2}$

Q. 99 If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

(A) 1
(B) $E/2$
(C) $2E$
(D) $4E$

2001 TWO MARKS

Q. 100 The impulse response functions of four linear systems S1, S2, S3, S4 are given respectively by

$$h_1(t) = 1,\ h_2(t) = u(t),\ h_3(t) = \frac{u(t)}{t+1} \text{ and } h_4(t) = e^{-3t}u(t)$$

where $u(t)$ is the unit step function. Which of these systems is time invariant, causal, and stable?

(A) S1
(B) S2
(C) S3
(D) S4

2000 ONE MARK

Q. 101 Given that $L[f(t)] = \frac{s+2}{s^2+1}$, $L[g(t)] = \frac{s^2+1}{(s+3)(s+2)}$ and $h(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$. $L[h(t)]$ is

(A) $\frac{s^2+1}{s+3}$
(B) $\frac{1}{s+3}$
(C) $\frac{s^2+1}{(s+3)(s+2)} + \frac{s+2}{s^2+1}$
(D) None of the above

Q. 102 The Fourier Transform of the signal $x(t) = e^{-3t^2}$ is of the following form, where A and B are constants :

(A) $Ae^{-B|f|}$
(B) Ae^{-Bf^2}
(C) $A + B|f|^2$
(D) Ae^{-Bf}

Q. 103 A system with an input $x(t)$ and output $y(t)$ is described by the relations : $y(t) = tx(t)$. This system is

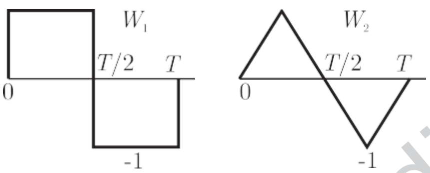
(A) linear and time - invariant
(B) linear and time varying
(C) non - linear and time - invariant
(D) non - linear and time - varying

Q. 104 A linear time invariant system has an impulse response e^{2t}, $t > 0$. If the initial conditions are zero and the input is e^{3t}, the output for $t > 0$ is

(A) $e^{3t} - e^{2t}$
(B) e^{5t}
(C) $e^{3t} + e^{2t}$
(D) None of these

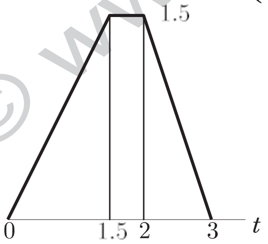
2000 TWO MARKS

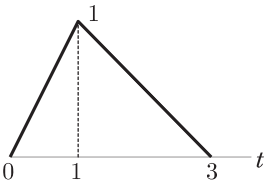
Q. 105 One period $(0, T)$ each of two periodic waveforms W_1 and W_2 are shown in the figure. The magnitudes of the n^{th} Fourier series coefficients of W_1 and W_2, for *n* $ 1, *n* odd, are respectively proportional to

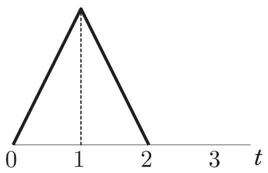


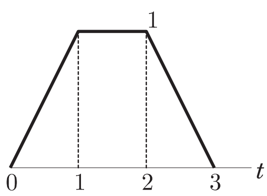
(A) $|n^{-3}|$ and $|n^{-2}|$
(B) $|n^{-2}|$ and $|n^{-3}|$
(C) $|n^{-1}|$ and $|n^{-2}|$
(D) $|n^{-4}|$ and $|n^{-2}|$

Q. 106 Let $u(t)$ be the step function. Which of the waveforms in the figure corresponds to the convolution of $u(t) - u(t-1)$ with $u(t) - u(t-2)$?

(A)


(B)


(C)


(D)


Q. 107 A system has a phase response given by $f(w)$, where w is the angular frequency. The phase delay and group delay at $w = w_0$ are respectively given by

(A) $-\frac{f(w_0)}{w_0}, -\frac{df(w)}{dw}\Big|_{w=w_0}$
(B) $f(w_o), -\frac{d^2 f(w_0)}{dw^2}\Big|_{w=w_o}$
(C) $\frac{w_o}{f(w_o)}, -\frac{df(w)}{d(w)}\Big|_{w=w_o}$
(D) $w_o f(w_o), \int_{-\infty}^{w_o} f(\lambda)$

1999 ONE MARK

Q. 108 The z-transform $F(z)$ of the function $f(nT) = a^{nT}$ is

(A) $\frac{z}{z - a^{T}}$ (B) $\frac{z}{z + a^{T}}$

(C) $\frac{z}{z - a^{-T}}$ (D) $\frac{z}{z + a^{-T}}$

Q. 109 If $[f(t)] = F(s)$, then $[f(t - T)]$ is equal to

(A) $e^{sT}F(s)$ (B) $e^{-sT}F(s)$

(C) $\frac{F(s)}{1 - e^{sT}}$ (D) $\frac{F(s)}{1 - e^{-sT}}$

Q. 110 A signal $x(t)$ has a Fourier transform $X(\omega)$. If $x(t)$ is a real and odd function of t, then $X(\omega)$ is

(A) a real and even function of ω

(B) a imaginary and odd function of ω

(C) an imaginary and even function of ω

(D) a real and odd function of ω

1999 TWO MARKS

Q. 111 The Fourier series representation of an impulse train denoted by

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) \text{ is given by}$$

(A) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp -\frac{j2\pi nt}{T_0}$ (B) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp -\frac{j\pi nt}{T_0}$

(C) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp \frac{j\pi nt}{T_0}$ (D) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp \frac{j2\pi nt}{T_0}$

Q. 112 The z-transform of a signal is given by $C(z) = \frac{1z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2}$. Its final value is

(A) 1/4 (B) zero

(C) 1.0 (D) infinity

1998 ONE MARK

Q. 113 If $F(s) = \frac{\omega}{s^2 + \omega^2}$, then the value of $\lim_{t \to \infty} f(t)$

(A) cannot be determined (B) is zero

(C) is unity (D) is infinite

Q. 114 The trigonometric Fourier series of a even time function can have only

(A) cosine terms (B) sine terms

(C) cosine and sine terms (D) d.c and cosine terms

Q. 115 A periodic signal $x(t)$ of period T_0 is given by $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}$

The dc component of $x(t)$ is

(A) $\frac{T_1}{T_0}$ (B) $\frac{T_1}{2T_0}$

(C) $\frac{2T_1}{T_0}$ (D) $\frac{T_0}{T_1}$

Q. 116 The unit impulse response of a linear time invariant system is the unit step function $u(t)$. For $t > 0$, the response of the system to an excitation $e^{-at}u(t), a > 0$ will be

(A) ae^{-at}

(B) $(1/a)(1 - e^{-at})$

(C) $a(1 - e^{-at})$

(D) $1 - e^{-at}$

Q. 117 The z-transform of the time function $\sum_{k=0}^{\infty} \delta(n - k)$ is

(A) $\frac{z-1}{z}$

(B) $\frac{z}{z-1}$

(C) $\frac{z}{(z-1)^2}$

(D) $\frac{(z-1)^2}{z}$

Q. 118 A distorted sinusoid has the amplitudes $A_1, A_2, A_3, ...$ of the fundamental, second harmonic, third harmonic,..... respectively. The total harmonic distortion is

(A) $\frac{A_2 + A_3 + ...}{A_1}$

(B) $\frac{A_2^2 + A_3^2 + ...}{A_1}$

(C) $\sqrt{\frac{A_2^2 + A_3^2 + ...}{A_1^2 + A_2^2 + A_3^2 + ...}}$

(D) $\left(\frac{A_2^2 + A_3^2 + ...}{A_1}\right)$

Q. 119 The Fourier transform of a function $x(t)$ is $X(f)$. The Fourier transform of $\frac{dX(t)}{df}$ will be

(A) $\frac{dX(f)}{df}$

(B) $j2\pi fX(f)$

(C) $jfX(f)$

(D) $\frac{X(f)}{jf}$

1997 ONE MARK

Q. 120 The function $f(t)$ has the Fourier Transform $g(\omega)$. The Fourier Transform

$$\mathcal{F}\{g(t)\} = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$ is

(A) $\frac{1}{2\pi} f(\omega)$

(B) $\frac{1}{2\pi} f(-\omega)$

(C) $2\pi f(-\omega)$

(D) None of the above

Q. 121 The Laplace Transform of $e^{\alpha t}\cos(\alpha t)$ is equal to

(A) $\frac{(s-\alpha)}{(s-\alpha)^2 + \alpha^2}$

(B) $\frac{(s+\alpha)}{(s-\alpha)^2 + \alpha^2}$

(C) $\frac{1}{(s-\alpha)^2}$

(D) None of the above

1996 ONE MARK

Q. 122 The trigonometric Fourier series of an even function of time does not have the

(A) dc term

(B) cosine terms

(C) sine terms

(D) odd harmonic terms

Q. 123 The Fourier transform of a real valued time signal has

(A) odd symmetry

(B) even symmetry

(C) conjugate symmetry

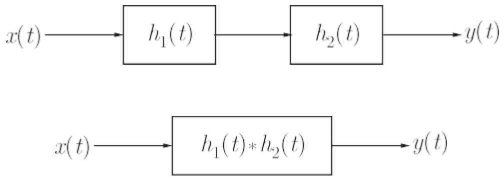
(D) no symmetry

****

SOLUTIONS

Sol. 1 Option (C) is correct.

If the two systems with impulse response $h_1(t)$ and $h_2(t)$ are connected in cascaded configuration as shown in figure, then the overall response of the system is the convolution of the individual impulse responses.



Sol. 2 Option (C) is correct.

Given, the input $x(t) = u(t-1)$

It's Laplace transform is

$$X(s) = \frac{e^{-s}}{s}$$

The impulse response of system is given

$$h(t) = t\,u(t)$$

Its Laplace transform is

$$H(s) = \frac{1}{s^2}$$

Hence, the overall response at the output is

$$Y(s) = X(s)H(s) = \frac{e^{-s}}{s^3}$$

Its inverse Laplace transform is

$$y(t) = \frac{(t-1)^2}{2}u(t-1)$$

Sol. 3 Option (A) is correct.

Given, the signal

$$v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \frac{\pi}{4}\right)$$

So we have

$$\omega_1 = 100 \text{ rad/s}, \ \omega_2 = 300 \text{ rad/s} \ \text{ and } \omega_3 = 500 \text{ rad/s}$$

Therefore, the respective time periods are

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{100}\text{ sec}, \ T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{300}\text{ sec and } T_3 = \frac{2\pi}{500}\text{ sec}$$

So, the fundamental time period of the signal is

$$\text{L.C.M.}(T_1, T_2, T_3) = \frac{LCM(2\pi, 2\pi, 2\pi)}{HCF(100, 300, 500)}$$

or, $$T_0 = \frac{2\pi}{100}$$

Hence, the fundamental frequency in rad / sec is $\omega_0 = \frac{2\pi}{10} = 100$ rad /s

Sol. 4 Option (A) is correct.

Given, the maximum frequency of the band-limited signal

$$f_m = 5 \text{ kHz}$$

According to the Nyquist sampling theorem, the sampling frequency must be greater than the Nyquist frequency which is given as

$$f_N = 2f_m = 2 \times 5 = 10 \text{ kHz}$$

So, the sampling frequency f_s must satisfy

$$f_s \geq f_N$$

$$f_s \geq 10 \text{ kHz}$$

only the option (A) doesn't satisfy the condition therefore, 5 kHz is not a valid sampling frequency.

Sol. 5 Option (C) is correct.

For a system to be casual, the R.O.C of system transfer function $H(s)$ which is rational should be in the right half plane and to the right of the right most pole.

For the stability of LTI system. All poles of the system should lie in the left half of S -plane and no repeated pole should be on imaginary axis. Hence, options (A), (B), (D) satisfies an LTI system stability and causality both.

But, Option (C) is not true for the stable system as, $|S| = 1$ have one pole in right hand plane also.

Sol. 6 Option (B) is correct.

The Laplace transform of unit step fun^n is

$$U(s) = \frac{1}{s}$$

So, the O / P of the system is given as

$$Y(s) = \left(\frac{1}{s}\right)\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

For zero initial condition, we check

$$u(t) = \frac{dy(t)}{dt}$$

$\Rightarrow$ $$U(s) = SY(s) - y(0)$$

$\Rightarrow$ $$U(s) = s\left(\frac{1}{s^2}\right) - y(0)$$

or, $$U(s) = \frac{1}{s} \qquad (y(0) = 0)$$

Hence, the O / P is correct which is

$$Y(s) = \frac{1}{s^2}$$

its inverse Laplace transform is given by

$$y(t) = tu(t)$$

Sol. 7 No Option is correct.

The matched filter is characterized by a frequency response that is given as

$$H(f) = G^*(f)\exp(-j2\pi fT)$$

where $g(t) \xrightarrow{f} G(f)$

Now, consider a filter matched to a known signal $g(t)$. The fourier transform of the resulting matched filter output $g_0(t)$ will be

$$G_0(f) = H(f)G(f) = G^*(f)G(f)\exp(-j2\pi fT)$$

$$= |G(f)|^2\exp(-j2\pi fT)$$

T is duration of $g(t)$

Assume $\exp(-j2\pi fT) = 1$

So, $G_0(f) = |G(f)|^2$

Since, the given Gaussian function is

$$g(t) = e^{-\pi t^2}$$

Fourier transform of this signal will be

$$g(t) = e^{-\pi t^2} \xrightarrow{f} e^{-\pi f^2} = G(f)$$

Therefore, output of the matched filter is

$$G_0(f) = \left|e^{-\pi f^2}\right|^2$$

Sol. 8 Option (B) is correct.

Given, the impulse response of continuous time system

$$h(t) = \delta(t-1) + \delta(t-3)$$

From the convolution property, we know

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

So, for the input

$$x(t) = u(t) \quad (\text{Unit step fun}^n)$$

The output of the system is obtained as

$$y(t) = u(t) * h(t) = u(t) * [\delta(t-1) + \delta(t-3)]$$

$$= u(t-1) + u(t-3)$$

At $t = 2$ $\quad y(2) = u(2-1) + u(2-3) = 1$

Sol. 9 Option (B) is correct.

Given, the differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = x(t)$$

Taking its Laplace transform with zero initial conditions, we have

$$s^2Y(s) + 5sY(s) + 6Y(s) = X(s) \quad \text{...(1)}$$

Now, the input signal is

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

i.e., $\quad x(t) = u(t) - u(t-2)$

Taking its Laplace transform, we obtain

$$X(s) = \frac{1}{s} - \frac{e^{-2s}}{s} = \frac{1-e^{-2s}}{s}$$

Substituting it in equation (1), we get

$$Y(s) = \frac{X(s)}{s^2+5s+6} = \frac{1-e^{2s}}{s(s^2+5s+6)} = \frac{1-e^{-2s}}{s(s+2)(s+3)}$$

Sol. 10 Option (D) is correct.

The solution of a system described by a linear, constant coefficient, ordinary, first order differential equation with forcing function $x(t)$ is $y(t)$, we can define a function relating $x(t)$ and $y(t)$ as below

$$P\frac{dy}{dt} + Qy + K = x(t)$$

where P, Q, K are constant. Taking the Laplace transform both the sides, we get

$$PsY(s) - Py(0) + QY(s) = X(s) \quad \text{...(1)}$$

Now, the solutions becomes

$$y_1(t) = -2y(t)$$

or, $Y_1(s) = -2Y(s)$

So, Eq. (1) changes to

$$PsY_1(s) - Py_1(0) + QY_1(s) = X_1(s)$$

or, $-2PsY(s) - Py_1(0) - 2QY(s) = X_1(s)$ .... (2)

Comparing Eq. (1) and (2), we conclude that

$$X_1(s) = -2X(s)$$

$$y_1(0) = -2y(0)$$

Which makes the two equations to be same. Hence, we require to change the initial condition to $-2y(0)$ and the forcing equation to $-2x(t)$

Sol. 11 Option (A) is correct.

Given, the DFT of vector $[a\ b\ c\ d]$ as

$$\text{D.F.T.}\{[a\ b\ c\ d]\} = [\alpha\ \beta\ \gamma\ \delta]$$

Also, we have

$$[p\ q\ r\ s] = [a\ b\ c\ d]\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix} \qquad ...(1)$$

For matrix circular convolution, we know

$$x[n] * h[n] = \begin{bmatrix} h_0 & h_2 & h_1 \\ h_1 & h_0 & h_2 \\ h_2 & h_1 & h_0 \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_1 \end{bmatrix}$$

where $\{x_0, x_1, x_2\}$ are three point signals for $x[n]$ and similarly for $h[n]$, h_0, h_1 and h_2 are three point signals. Comparing this transformation to Eq(1), we get

$$[p\ q\ r\ s] = \begin{bmatrix} a & d & c \\ b & a & d \\ c & b & a \\ d & c & b \end{bmatrix}^T [a\ b\ c\ d]$$

$$= [a\ b\ c\ d]^T * [a\ b\ c\ d]^T$$

$$= \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Now, we know that

$$x_1[n] * x_2[n] = X_{1DFT}[k]\, X_{2,DFT}[k]$$

So, $$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix} * \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix}$$

$$= \{\alpha^2\ \beta^2\ \gamma^2\ \delta^2\}$$

Sol. 12 Option (D) is correct.

Using s-domain differentiation property of Laplace transform.

If $$f(t) \overset{L}{\longleftrightarrow} F(s)$$

$$tf(t) \overset{L}{\longleftrightarrow} -\frac{dF(s)}{ds}$$

So, $$\mathcal{L}[tf(t)] = \frac{-d}{ds}\left[\frac{1}{s^2+s+1}\right] = \frac{2s+1}{(s^2+s+1)^2}$$

Sol. 13 Option (C) is correct.

$$x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u(n)$$

Taking z-transform

$$X[z] = \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n z^{-n}u[n] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{-n} z^{-n}u[-n-1] - \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}u[n]$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1}\left(\frac{1}{3}\right)^{-n} z^{-n} - \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}$$

$$= \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty}\left(\frac{1}{3}z\right)^m}_{\text{II}} - \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n}_{\text{III}} \qquad \text{Taking } m = -n$$

Series I converges if $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

Series II converges if $\left|\frac{1}{3}z\right| < 1$ or $|z| < 3$

Series III converges if $\left|\frac{1}{2z}\right| < 1$ or $|z| > \frac{1}{2}$

Region of convergence of $X(z)$ will be intersection of above three

So, ROC : $\frac{1}{2} < |z| < 3$

Sol. 14 Option (D) is correct.

$$y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

Time Invariance :

Let, $x(t) = \delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta(\tau)\cos(3\tau)\,d\tau = u(t)\cos(0) = u(t)$$

For a delayed input $(t - t_0)$ output is

$$y(t, t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)\cos(3\tau)\,d\tau = u(t)\cos(3t_0)$$

Delayed output,

$$y(t - t_0) = u(t - t_0)$$

$$y(t, t_0) \neq y(t - t_0)$$ System is not time invariant.

Stability :

Consider a bounded input $x(t) = \cos 3t$

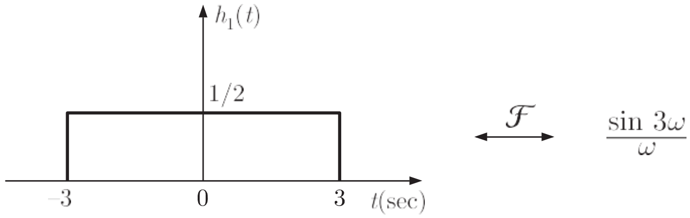
$$y(t) = \int_{-\infty}^{t} \cos^2 3\tau\,d\tau = \int_{-\infty}^{t} \frac{1-\cos 6\tau}{2}\,d\tau = \frac{1}{2}\int_{-\infty}^{t} 1\,d\tau - \frac{1}{2}\int_{-\infty}^{t} \cos 6\tau\,d\tau$$

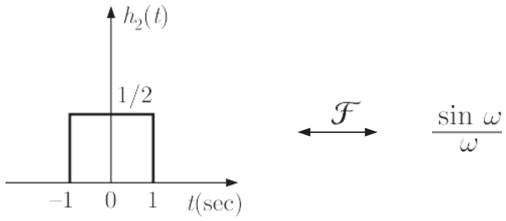
As $t \to \infty$, $y(t) \to \infty$ (unbounded) System is not stable.

Sol. 15 Option (C) is correct.

$$H(j\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega} = \frac{\sin 3\omega}{\omega} + \frac{\sin \omega}{\omega}$$

We know that inverse Fourier transform of sin c function is a rectangular function.





So, inverse Fourier transform of $H(jw)$

$$h(t) = h_1(t) + h_2(t)$$
$$h(0) = h_1(0) + h_2(0) = \frac{1}{2} + \frac{1}{2} = 1$$

Sol. 16 Option (A) is correct.

Convolution sum is defined as

$$y[n] = h[n] * g[n] = \sum_{k=-3}^{3} h[n]g[n-k]$$

For causal sequence, $y[n] = \sum_{k=0}^{3} h[n]\,g[n-k]$

$$y[n] = h[n]\,g[n] + h[n]g[n-1] + h[n]g[n-2] + ...$$

For $n = 0$, $y[0] = h[0]g[0] + h[1]g[-1] + ...$

$$y[0] = h[0]g[0] \qquad g[-1] = g[-2] = ...0$$
$$y[0] = h[0]g[0] \qquad ...(i)$$

For $n = 1$, $y[1] = h[1]g[1] + h[1]g[0] + h[1]g[-1] + ...$

$$y[1] = h[1]\,g[1] + h[1]\,g[0]$$
$$\frac{1}{2} = \frac{1}{2}g[1] + \frac{1}{2}g[0] \qquad h[1] = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$
$$1 = g[1] + g[0]$$
$$g[1] = 1 - g[0]$$

From equation (i), $g[0] = \frac{y[0]}{h[0]} = \frac{1}{1} = 1$

So, $g[1] = 1 - 1 = 0$

Sol. 17 Option (A) is correct.

We have $100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$

Applying Laplace transform we get

$$100s^2\,Y(s) - 20s\,Y(s) + Y(s) = X(s)$$

or $H(s) = \frac{Y(s)}{X(s)} = \frac{1}{100s^2 - 20s + 1}$

$$= \frac{1/100}{s^2 - (1/5)s + 1/100} = \frac{A}{s^2 + 2\xi\omega_n s + \omega^2}$$

Here $\omega_n = 1/10$ and $2\xi\omega_n = -1/5$ giving $\xi = -1$

Roots are $s = 1/10, 1/10$ which lie on Right side of s plane thus unstable.

Sol. 18 Option (C) is correct.

For an even function Fourier series contains dc term and cosine term (even and odd harmonics).

Sol. 19 Option (B) is correct.

Function $h(n) = a^n u(n)$ stable if $|a| < 1$ and Unstable if $|a| \geq 1$ We have

$h(n) = 2^n u(n-2);$

Here $|a| = 2$ therefore $h(n)$ is unstable and since $h(n) = 0$ for $n < 0$

Therefore $h(n)$ will be causal. So $h(n)$ is causal and not stable.

Sol. 20 Option (A) is correct.

$$\text{Impulse response} = \frac{d}{dt}(\text{step response})$$

$$= \frac{d}{dt}(1 - e^{-\alpha t}) = 0 + \alpha e^{-\alpha t} = \alpha e^{-\alpha t}$$

Sol. 21 Option (D) is correct.

We have $x(t) = \exp(-2t)\mu(t) + \delta(t-6)$ and $h(t) = u(t)$

Taking Laplace Transform we get

$$X(s) = \left(\frac{1}{s+2} + e^{-6s}\right) \text{ and } H(s) = \frac{1}{s}$$

Now $Y(s) = H(s)X(s)$

$$= \frac{1}{s}\left[\frac{1}{s+2} + e^{-6s}\right] = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}$$

or $$Y(s) = \frac{1}{2s} - \frac{1}{2(s+2)} + \frac{e^{-6s}}{s}$$

Thus $y(t) = 0.5[1 - \exp(-2t)]u(t) + u(t-6)$

Sol. 22 Option (B) is correct.

$$y(n) = x(n-1)$$

or $$Y(z) = z^{-1}X(z)$$

or $$\frac{Y(z)}{X(z)} = H(z) = z^{-1}$$

Now $$H_1(z)H_2(z) = z^{-1}$$

$$\left(\frac{1 - 0.4z^{-1}}{1 - 0.6z^{-1}}\right)H_2(z) = z^{-1}$$

$$H_2(z) = \frac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$$

Sol. 23 Option (B) is correct.

For 8 point DFT, $x^*[1] = x[7]; x^*[2] = x[6]; x^*[3] = x[5]$ and it is conjugate symmetric about $x[4]$, $x[6] = 0$; $x[7] = 1 + j3$

Sol. 24 Option (A) is correct.

We know that $$az^{-a} \xleftrightarrow{\text{Inverse Z-transform}} a\delta[n-a]$$

Given that $$X(z) = 5z^2 + 4z^{-1} + 3$$

Inverse z-transform $$x[n] = 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$$

Sol. 25 Option (C) is correct.

For a function $x(t)$ trigonometric fourier series is

$$x(t) = A_o + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

Where, $A_o = \frac{1}{T}\int_{T_0} x(t)\,dt$ $\qquad T_0 \to$ fundamental period

and $A_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega t\,dt$

$$B_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega t\,dt$$

For an even function $x(t)$, $B_n = 0$

Since given function is even function so coefficient $B_n = 0$, only cosine and constant terms are present in its fourier series representation

Constant term $A_0 = \frac{1}{T}\int_{-T/4}^{3T/4} x(t)\,dt = \frac{1}{T}\left[\int_{-T/4}^{T/4} A\,dt + \int_{T/4}^{3T/4} -2A\,dt\right]$

$$= \frac{1}{T}\left[\frac{TA}{2} - 2A\frac{T}{2}\right] = -\frac{A}{2}$$

Constant term is negative.

Sol. 26 Option (C) is correct.

We have $h_1[n] = \delta[n-1]$ or $H_1[Z] = Z^{-1}$

and $h_2[n] = \delta[n-2]$ or $H_2(Z) = Z^{-2}$

Response of cascaded system

$$H(z) = H_1(z) \cdot H_2(z) = z^{-1} \cdot z^{-2} = z^{-3}$$

or, $h[n] = \delta[n-3]$

Sol. 27 Option (D) is correct.

For an N-point FET algorithm butterfly operates on one pair of samples and involves two complex addition and one complex multiplication.

Sol. 28 Option (D) is correct.

We have $f(t) = \mathcal{L}^{-1}\left[\frac{3s+1}{s^3 + 4s^2 + (k-3)s}\right]$

and $\lim_{t \to \infty} f(t) = 1$

By final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) = 1$$

or $\lim_{s \to 0} \frac{s.(3s+1)}{s^3 + 4s^2 + (k-3)s} = 1$

or $\lim_{s \to 0} \frac{s(3s+1)}{s[s^2 + 4s + (k-3)]} = 1$

$$\frac{1}{k-3} = 1$$

or $k = 4$

Sol. 29 Option (B) is correct.

System is described as

$$\frac{d^2y(t)}{dt^2} + 4\frac{dt(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Taking Laplace transform on both side of given equation

$$s^2Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s)$$

$$(s^2 + 4s + 3)Y(s) = 2(s+2)X(s)s$$

Transfer function of the system

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2(s+2)}{s^2+4s+3} = \frac{2(s+2)}{(s+3)(s+1)}$$

Input $\quad x(t) = e^{-2t}u(t)$

or, $\quad X(s) = \frac{1}{(s+2)}$

Output $\quad Y(s) = H(s) \cdot X(s) = \frac{2(s+2)}{(s+3)(s+1)} \cdot \frac{1}{(s+2)}$

By Partial fraction

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+3}$$

Taking inverse Laplace transform

$$y(t) = (e^{-t} - e^{-3t})u(t)$$

Sol. 30 Option (C) is correct.

We have $\quad H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

By partial fraction $H(z)$ can be written as

$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{1}{\left(1 - \frac{1}{4}z^{-1}\right)}$$

For ROC : $|z| > 1/2$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n], n > 0 \qquad \frac{1}{1-z^{-1}} = a^n u[n], |z| > a$$

Thus system is causal. Since ROC of $H(z)$ includes unit circle, so it is stable also.
Hence S_1 is True

For ROC : $|z| < \frac{1}{4}$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{4}\right)^n u(n), \ |z| > \frac{1}{4}, |z| < \frac{1}{2}$$

System is not causal. ROC of $H(z)$ does not include unity circle, so it is not stable and S_3 is True

Sol. 31 Option (A) is correct.

The Fourier series of a real periodic function has only cosine terms if it is even and sine terms if it is odd.

Sol. 32 Option (B) is correct.

Given function is

$$f(t) = \sin^2 t + \cos 2t = \frac{1-\cos 2t}{2} + \cos 2t = \frac{1}{2} + \frac{1}{2}\cos 2t$$

The function has a DC term and a cosine function. The frequency of cosine terms is

$$w = 2 = 2pf \Rightarrow f = \frac{1}{p} \text{ Hz}$$

The given function has frequency component at 0 and $\frac{1}{p}$ Hz.

Sol. 33 Option (A) is correct.

$$x[n] = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$$

Taking z transform we have

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{3}z^{-1}\right)^n - \sum_{n=-\infty}^{-1}\left(\frac{1}{2}z^{-1}\right)^n$$

First term gives $\frac{1}{3}z^{-1} < 1 \Rightarrow \frac{1}{3} < |z|$

Second term gives $\frac{1}{2}z^{-1} > 1 \Rightarrow \frac{1}{2} > |z|$

Thus its ROC is the common ROC of both terms. that is

$$\frac{1}{3} < |z| < \frac{1}{2}$$

Sol. 34 Option (B) is correct.

By property of unilateral Laplace transform

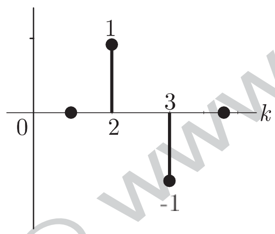
$$\int_{-\infty}^{t} f(\tau)\,d\tau \xleftrightarrow{L} \frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0^-} f(\tau)\,d\tau$$

Here function is defined for $0 < \tau < t$, Thus

$$\int_0^t f(\tau) \xleftrightarrow{L} \frac{F(s)}{s}$$

Sol. 35 Option (A) is correct.

We have $h(2) = 1$, $h(3) = -1$ otherwise $h(k) = 0$. The diagram of response is as follows :



It has the finite magnitude values. So it is a finite impulse response filter. Thus S_2 is true but it is not a low pass filter. So S_1 is false.

Sol. 36 Option (B) is correct.

Here $h(t) \neq 0$ for $t < 0$. Thus system is non causal. Again any bounded input $x(t)$ gives bounded output $y(t)$. Thus it is BIBO stable.

Here we can conclude that option (B) is correct.

Sol. 37 Option (D) is correct.

We have $x[n] = \{1, 0, 2, 3)$ and $N = 4$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N} \quad k = 0, 1\ldots N-1$$

For $N = 4$, $$X[k] = \sum_{n=0}^{3} x[n]e^{-j2\pi nk/4} \quad k = 0, 1, \ldots 3$$

Now $$X[0] = \sum_{n=0}^{3} x[n] = x[0] + x[1] + x[2] + x[3] = 1 + 0 + 2 + 3 = 6$$

$$x[1] = \sum_{n=0}^{3} x[n]e^{-j\pi n/2} = x[0] + x[1]e^{-j\pi/2} + x[2]e^{-j\pi} + x[3]e^{-j\pi 3/2}$$

$$= 1 + 0 - 2 + j3 = -1 + j3$$

$$X[2] = \sum_{n=0}^{3} x[n]e^{-j\pi n} = x[0] + x[1]e^{-j\pi} + x[2]e^{-j2\pi} + x[3]e^{-j\pi 3}$$

$$= 1 + 0 + 2 - 3 = 0$$

$$X[3] = \sum_{n=0}^{3} x[n]e^{-j3\pi n/2} = x[0] + x[1]e^{-j3\pi/2} + x[2]e^{-j3\pi} + x[3]e^{-j9\pi/2}$$

$$= 1 + 0 - 2 - j3 = -1 - j3$$

Thus $[6, -1+j3, \ 0, -1-j3]$

Sol. 38 Option (A) is correct.

Sol. 39 Option (C) is correct.

The output of causal system depends only on present and past states only. In option (A) $y(0)$ depends on $x(-2)$ and $x(4)$.

In option (B) $y(0)$ depends on $x(1)$. In option (C) $y(0)$ depends on $x(-1)$. In option (D) $y(0)$ depends on $x(5)$.

Thus only in option (C) the value of $y(t)$ at $t = 0$ depends on $x(-1)$ past value. In all other option present value depends on future value.

Sol. 40 Option (D) is correct.

We have $\quad h(t) = e^{at}u(t) + e^{bt}u(-t)$

This system is stable only when bounded input has bounded output For stability $at < 0$ for $t > 0$ that implies $a < 0$ and $bt > 0$ for $t > 0$ that implies $b > 0$. Thus, a is negative and b is positive.

Sol. 41 Option (C) is correct.

$$G(s) = \frac{K(s+1)}{(s+2)(s+4)}, \text{ and } R(s) = \frac{1}{s}$$

$$C(s) = G(s)R(s) = \frac{K(s+1)}{s(s+2)(s+4)}$$

$$= \frac{K}{8s} + \frac{K}{4(s+2)} - \frac{3K}{8(s+4)}$$

Thus $\quad c(t) = K\left[\frac{1}{8} + \frac{1}{4}e^{-2t} - \frac{3}{8}e^{-4t}\right]u(t)$

At steady-state, $c(\infty) = 1$

Thus $\quad \frac{K}{8} = 1$ or $K = 8$

Then, $\quad G(s) = \frac{8(s+1)}{(s+2)(s+4)} = \frac{12}{(s+4)} - \frac{4}{(s+2)}$

$$h(t) = L^{-1}G(s) = (-4e^{-2t} + 12e^{-4t})u(t)$$

Sol. 42 Option (A) is correct.

We have $\quad x(t) = \begin{cases} 1 & \text{for } -1 \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$

Fourier transform is

$$\int_{-\infty}^{\infty} e^{-j\omega t}x(t)dt = \int_{-1}^{1} e^{-j\omega t}1dt = \frac{1}{-j\omega}[e^{-j\omega t}]_{-1}^{1}$$

$$= \frac{1}{-j\omega}(e^{-j\omega} - e^{j\omega}) = \frac{1}{-j\omega}(-2j\sin\omega) = \frac{2\sin\omega}{\omega}$$

This is zero at $\omega = \pi$ and $\omega = 2\pi$

Sol. 43 Option (D) is correct.

Given $\quad h(n) = [1, -1, 2]$

$$x(n) = [1, 0, 1]$$
$$y(n) = x(n) * h(n)$$

The length of $y[n]$ is $= L_1 + L_2 - 1 = 3 + 3 - 1 = 5$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$
$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$$
$$= x(0)\, h(2-0) + x(1)\, h(2-1) + x(2)\, h(2-2)$$
$$= h(2) + 0 + h(0) = 1 + 2 = 3$$

There are 5 non zero sample in output sequence and the value of $y[2]$ is 3.

Sol. 44 Option (B) is correct.

Mode function are not linear. Thus $y(t) = |x(t)|$ is not linear but this functions is time invariant. Option (A) and (B) may be correct.

The $y(t) = t|x(t)|$ is not linear, thus option (B) is wrong and (a) is correct. We can see that

R_1: $y(t) = t^2 x(t)$ Linear and time variant.

R_2: $y(t) = t|x(t)|$ Non linear and time variant.

R_3: $y(t) = |x(t)|$ Non linear and time invariant

R_4: $y(t) = x(t-5)$ Linear and time invariant

Sol. 45 Option (A) is correct.

Given : $$y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\, x(n+r)$$

It is Auto correlation.

Hence $$y(n) = r_{xx}(n) \xleftrightarrow{DFT} |X(k)|^2$$

Sol. 46 Option (B) is correct.

Current through resistor (i.e. capacitor) is

$$I = I(0^+)\, e^{-t/RC}$$

Here, $$I(0^+) = \frac{V}{R} = \frac{5}{200k} = 25m\text{A}$$

$$RC = 200k \times 10m = 2 \text{ sec}$$
$$I = 25e^{-\frac{t}{2}}\, m\text{A} = V_R \times R = 5e^{-\frac{t}{2}} \text{ V}$$

Here the voltages across the resistor is input to sampler at frequency of 10 Hz. Thus

$$x(n) = 5e^{\frac{-n}{2 \times 10}} = 5e^{-0.05n} \quad \text{For } t > 0$$

Sol. 47 Option (C) is correct.

Since $$x(n) = 5e^{-0.05n}\, u(n) \text{ is a causal signal}$$

Its z transform is $$X(z) = 5\left[\frac{1}{1 - e^{-0.05} z^{-1}}\right] = \frac{5z}{z - e^{-0.05}}$$

Its ROC is $|e^{-0.05} z^{-1}| > 1 \Rightarrow |z| > e^{-0.05}$

Sol. 48 Option (C) is correct.

$$h(t) = e^{-2t}\, u(t)$$
$$H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-2t} e^{-j\omega t} dt = \int_0^{\infty} e^{-(2+j\omega)t} dt = \frac{1}{(2+j\omega)}$$

Sol. 49 Option (D) is correct.

$$H(j\omega) = \frac{1}{(2+j\omega)}$$

The phase response at $\omega = 2$ rad / sec is

$$\angle H(j\omega) = -\tan^{-1}\frac{\omega}{2} = -\tan^{-1}\frac{2}{2} = -\frac{\pi}{4} = -0.25\pi$$

Magnitude response at $\omega = 2$ rad / sec is

$$|H(j\omega)| = \frac{1}{\sqrt{2^2+\omega^2}} = \frac{1}{2\sqrt{2}}$$

Input is $x(t) = 2\cos(2t)$

Output is

$$= \frac{1}{2\sqrt{2}} \times 2\cos(2t - 0.25\pi)$$

$$= \frac{1}{\sqrt{2}}\cos[2t - 0.25\pi]$$

Sol. 50 Option (D) is correct.

$$Y(s) = \frac{1}{s(s-1)}$$

Final value theorem is applicable only when all poles of system lies in left half of S-plane. Here $s = 1$ is right s-plane pole. Thus it is unbounded.

Sol. 51 Option (A) is correct.

$$x(t) = e^{-t}u(t)$$

Taking Fourier transform

$$X(j\omega) = \frac{1}{1+j\omega}$$

$$|X(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

Magnitude at 3dB frequency is $\frac{1}{\sqrt{2}}$

Thus $$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\omega^2}}$$

or $\omega = 1$ rad

or $f = \frac{1}{2\pi}$ Hz

Sol. 52 Option (B) is correct.

For discrete time Fourier transform (DTFT) when $N \to \infty$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

Putting $n = 0$ we get

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega 0} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$

or $$\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega = 2\pi x[0] = 2\pi \times 5 = 10\pi$$

Sol. 53 Option (B) is correct.

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Since ROC includes unit circle, it is left handed system

$$x(n) = -(0.5)(2)^{-n} u(-n-1)$$
$$x(0) = 0$$

If we apply initial value theorem

$$x(0) = \lim_{z \to \infty} X(z) = \lim_{z \to \infty} \frac{0.5}{1 - 2z^{-1}} = 0.5$$

That is wrong because here initial value theorem is not applicable because signal $x(n)$ is defined for $n < 0$.

Sol. 54 Option (A) is correct.

A Hilbert transformer is a non-linear system.

Sol. 55 Option (B) is correct.

$$H(f) = \frac{5}{1 + j10\pi f}$$
$$H(s) = \frac{5}{1 + 5s} = \frac{5}{5\left(s + \frac{1}{5}\right)} = \frac{1}{s + \frac{1}{5}}$$

Step response $\quad Y(s) = \frac{1}{s\left(s + \frac{1}{5}\right)} = \frac{1}{s}\frac{1}{\left(s + \frac{1}{5}\right)} = \frac{5}{s} - \frac{5}{s + \frac{1}{5}}$

or $\quad y(t) = 5(1 - e^{-t/5})u(t)$

Sol. 56 Option (A) is correct.

$$x(t) \xleftrightarrow{F} X(j\omega)$$

Using scaling we have

$$x(5t) \xleftrightarrow{F} \frac{1}{5} X\left(\frac{j\omega}{5}\right)$$

Using shifting property we get

$$x\left[5\left(t - \frac{3}{5}\right)\right] \xleftrightarrow{F} \frac{1}{5} X\left(\frac{j\omega}{5}\right) e^{-\frac{j3\omega}{5}}$$

Sol. 57 Option (D) is correct.

Dirac delta function $\delta(t)$ is defined at $t = 0$ and it has infinite value a $t = 0$. The area of dirac delta function is unity.

Sol. 58 Option (D) is correct.

The ROC of addition or subtraction of two functions $x_1(n)$ and $x_2(n)$ is $R_1 + R_2$. We have been given ROC of addition of two function and has been asked ROC of subtraction of two function. It will be same.

Sol. 59 Option (A) is correct.

As we have $\quad x(t) = \sin t, \quad$ thus $\omega = 1$

Now $\quad H(s) = \frac{1}{s + 1}$

or $\quad H(j\omega) = \frac{1}{j\omega + 1} = \frac{1}{j + 1}$

or $\quad H(j\omega) = \frac{1}{\sqrt{2}} \angle -45^\circ$

Thus $\quad y(t) = \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$

Sol. 60 Option (C) is correct.

$$F(s) = \frac{\omega_0}{s^2 + \omega^2}$$
$$L^{-1}F(s) = \sin \omega_o t$$
$$f(t) = \sin \omega_o t$$

Thus the final value is $-1 \le f(\infty) \le 1$

Sol. 61 Option (C) is correct.

$$y(n) = \left[\sin\frac{5}{6}\pi n\right] x(n)$$

Let $x(n) = \delta(n)$

Now $y(n) = \sin 0 = 0$ (bounded) BIBO stable

Sol. 62 Option (B) is correct.

$$c(t) = 1 - e^{-2t}$$

Taking Laplace transform

$$C(s) = \frac{C(s)}{U(s)} = \frac{2}{s(s+2)} \times s = \frac{2}{s+2}$$

Sol. 63 Option (C) is correct.

$$h(t) = e^{-t} \xrightarrow{L} H(s) = \frac{1}{s+1}$$

$$x(t) = u(t) \xrightarrow{L} X(s) = \frac{1}{s}$$

$$Y(s) = H(s)X(s) = \frac{1}{s+1} \times \frac{1}{s} = \frac{1}{s} - \frac{1}{s+1}$$

$$y(t) = u(t) - e^{-t}$$

In steady state i.e. $t \to \infty$, $y(\infty) = 1$

Sol. 64 Option (C) is correct.

Fourier series is defined for periodic function and constant.

$3\sin(25t)$ is a periodic function.

$4\cos(20t+3) + 2\sin(710t)$ is sum of two periodic function and also a periodic function.

$e^{-|t|}\sin(25t)$ is not a periodic function, so FS can't be defined for it.

1 is constant

Sol. 65 Option (A) is correct.

$$\text{Ev}\{g(t)\} = \frac{g(t) + g(-t)}{2}$$

$$\text{odd}\{g(t)\} = \frac{g(t) - g(-t)}{2}$$

Here $g(t) = u(t)$

Thus $u_e(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}$

$$u_o(t) = \frac{u(t) - u(-t)}{2} = \frac{x(t)}{2}$$

Sol. 66 Option (C) is correct.

Here $x_1(n) = \left(\frac{5}{6}\right)^n u(n)$

$$X_1(z) = \frac{1}{1 - \left(\frac{5}{6}z^{-1}\right)} \qquad \text{ROC}: R_1 \to |z| > \frac{5}{6}$$

$$x_2(n) = -\left(\frac{6}{5}\right)^n u(-n-1)$$

$$X_1(z) = 1 - \frac{1}{1 - \left(\frac{6}{5}z^{-1}\right)} \qquad \text{ROC}: R_2 \to |z| < \frac{6}{5}$$

Thus ROC of $x_1(n) + x_2(n)$ is $R_1 + R_2$ which is $\frac{5}{6} < |z| < \frac{6}{5}$

Sol. 67 Option (D) is correct.

For causal system $h(t) = 0$ for $t \le 0$. Only (D) satisfy this condition.

Sol. 68 Option (D) is correct.

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$y(n) = x^2(n) = \left(\frac{1}{2}\right)^{2n} u^2(n)$$

or
$$y(n) = \left[\left(\frac{1}{2}\right)^2\right]^n u(n) = \left(\frac{1}{4}\right)^n u(n) \quad ...(1)$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{n=\infty} y(n) e^{-j\omega n} = \sum_{n=0}^{n=\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

or
$$Y(e^{j0}) = \sum_{n=0}^{n=\infty} \left(\frac{1}{4}\right)^n = 1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

Alternative :

Taking z transform of (1) we get

$$Y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

Substituting $z = e^{j\omega}$ we have

$$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j0}) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

Sol. 69 Option (A) is correct.

$$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right) + 4\sin 15\pi t$$

$$= 8\sin 20\pi t + 4\sin 15\pi t$$

Here $A_1 = 8$ and $A_2 = 4$. Thus power is

$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2} = \frac{8^2}{2} + \frac{4^2}{2} = 40$$

Sol. 70 Option (A) is correct.

$$y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d - T)$$

Taking Fourier transform we have

$$Y(\omega) = 0.5e^{-j\omega(-t_d+T)}X(\omega) + e^{-j\omega t_d}X(\omega) + 0.5e^{-j\omega(-t_d-T)}X(\omega)$$

or
$$\frac{Y(\omega)}{X(\omega)} = e^{-j\omega t_d}[0.5e^{j\omega T} + 1 + 0.5e^{-j\omega T}]$$

$$= e^{-j\omega t_d}[0.5(e^{j\omega T} + e^{-j\omega T}) + 1] = e^{-j\omega t_d}[\cos\omega T + 1]$$

or
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = e^{-j\omega t_d}(\cos\omega T + 1)$$

Sol. 71 Option (C) is correct.

For continuous and aperiodic signal Fourier representation is continuous and aperiodic.

For continuous and periodic signal Fourier representation is discrete and aperiodic.

For discrete and aperiodic signal Fourier representation is continuous and periodic.

For discrete and periodic signal Fourier representation is discrete and periodic.

Sol. 72 Option (B) is correct.

$$y(n) = Ax(n - n_o)$$

Taking Fourier transform

$$Y(e^{j\omega}) = Ae^{-j\omega_o n_o} X(e^{j\omega})$$

or $$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = Ae^{-j\omega_o n_o}$$

Thus $$\angle H(e^{j\omega}) = -\omega_o n_o$$

For LTI discrete time system phase and frequency of $H(e^{j\omega})$ are periodic with period 2π. So in general form

$$\theta(\omega) = -n_o\omega_o + 2\pi k$$

Sol. 73 Option (A) is correct.

From $$x(n) = [\tfrac{1}{2}, 1, 2, 1, 1, \tfrac{1}{2}]$$

$$y(n) = x\left(\frac{n}{2} - 1\right), n \text{ even}$$

$$= 0, \text{ for } n \text{ odd}$$

$n = -2$, $y(-2) = x(\frac{-2}{2} - 1) = x(-2) = \frac{1}{2}$

$n = -1$, $y(-1) = 0$

$n = 0$, $y(0) = x(\frac{0}{2} - 1) = x(-1) = 1$

$n = 1$, $y(1) = 0$

$n = 2$ $y(2) = x(\frac{2}{2} - 1) = x(0) = 2$

$n = 3$, $y(3) = 0$

$n = 4$ $y(4) = x(\frac{4}{2} - 1) = x(1) = 1$

$n = 5$, $y(5) = 0$

$n = 6$ $y(6) = x(\frac{6}{2} - 1) = x(2) = \frac{1}{2}$

Hence $$y(n) = \frac{1}{2}\delta(n + 2) + \delta(n) + 2\delta(n - 2) + \delta(n - 4) + \frac{1}{2}\delta(n - 6)$$

Sol. 74 Option (C) is correct.

Here $y(n)$ is scaled and shifted version of $x(n)$ and again $y(2n)$ is scaled version of $y(n)$ giving

$$z(n) = y(2n) = x(n - 1)$$

$$= \frac{1}{2}\delta(n + 1) + \delta(n) + 2\delta(n - 1) + \delta(n - 2) + \frac{1}{2}\delta(n - 3)$$

Taking Fourier transform.

$$Z(e^{j\omega}) = \frac{1}{2}e^{j\omega} + 1 + 2e^{-j\omega} + e^{-2j\omega} + \frac{1}{2}e^{-3j\omega}$$

$$= e^{-j\omega}\left(\frac{1}{2}e^{2j\omega} + e^{j\omega} + 2 + e^{-j\omega} + \frac{1}{2}e^{-2j\omega}\right)$$

$$= e^{-j\omega}\left(\frac{e^{2j\omega} + e^{-2j\omega}}{2} + e^{j\omega} + 2 + e^{-j\omega}\right)$$

or $$Z(e^{j\omega}) = e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$$

Sol. 75 Option (B) is correct.

$$x(t) \xleftrightarrow{F} X(f)$$

Using scaling we have

$$x(at) \xleftrightarrow{F} \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

Thus $x\left(\frac{1}{3}f\right) \xleftrightarrow{F} 3X(3f)$

Using shifting property we get

$$e^{-j2\pi f_0 t} x(t) = X(f + f_0)$$

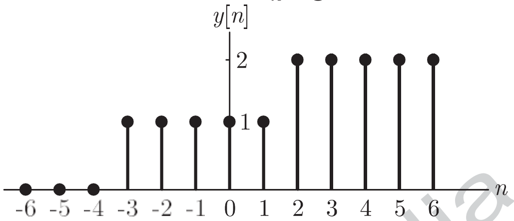
Thus $\frac{1}{3} e^{-j\frac{4}{3}\pi t} x\left(\frac{1}{3}t\right) \xleftrightarrow{F} X(3f + 2)$

$$e^{-j2\pi\frac{2}{3}t} x\left(\frac{1}{3}t\right) \xleftrightarrow{F} 3X(3(f + \tfrac{2}{3}))$$

$$\frac{1}{3} e^{-j\pi\frac{4}{3}t} x\left(\frac{1}{3}t\right) \xleftrightarrow{F} X[3(f + \tfrac{2}{3})]$$

Sol. 76 Option (A) is correct.

A system is stable if $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$. The plot of given $h(n)$ is



Thus $\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-3}^{6} |h(n)|$

$$= 1 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2$$

$$= 15 < \infty$$

Hence system is stable but $h(n) \neq 0$ for $n < 0$. Thus it is not causal.

Sol. 77 Option (D) is correct.

$$H(z) = \frac{z}{z - 0.2} \qquad |z| < 0.2$$

We know that

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1 - az^{-1}} \qquad |z| < a$$

Thus $h[n] = -(0.2)^n u[-n-1]$

Sol. 78 Option (C) is correct.

The Fourier transform of a conjugate symmetrical function is always real.

Sol. 79 Option (A) is correct.

We have $x(n) = [-4 - j5, \quad \underset{\uparrow}{1 + 2j}, \quad 4]$

$$x^*(n) = [-4 + j5, \quad \underset{\uparrow}{1 - 2j}, 4]$$

$$x^*(-n) = [4, \quad \underset{\uparrow}{1 - 2j}, \quad -4 + j5]$$

$$x_{cas}(n) = \frac{x(n) - x^*(-n)}{2} = [-4 - j\tfrac{5}{2}, \quad \underset{\uparrow}{2j} \quad 4 - j\tfrac{5}{2}]$$

Sol. 80 Option (C) is correct.

We have $2y(n) = \alpha y(n-2) - 2x(n) + \beta x(n-1)$

Taking z transform we get

$$2Y(z) = \alpha Y(z) z^{-2} - 2X(z) + \beta X(z) z^{-1}$$

or $$\frac{Y(z)}{X(z)} = \left(\frac{bz^{-1} - 2}{2 - az^{-2}}\right) \qquad ...(i)$$

or $$H(z) = \frac{z\left(\frac{b}{2} - z\right)}{\left(z^2 - \frac{a}{2}\right)}$$

It has poles at $\pm\sqrt{a/2}$ and zero at 0 and $b/2$. For a stable system poles must lie inside the unit circle of z plane. Thus

$$\therefore \left|\frac{a}{2}\right| < 1$$

or $$|a| < 2$$

But zero can lie anywhere in plane. Thus, b can be of any value.

Sol. 81 Option (D) is correct.

We have $$x(n) = e^{j\pi n/4}$$

and $$h(n) = 4\sqrt{2}\,\delta(n+2) - 2\sqrt{2}\,\delta(n+1) - 2\sqrt{2}\,\delta(n-1) + 4\sqrt{2}\,\delta(n-2)$$

Now $$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)h(k) = \sum_{k=-2}^{2} x(n-k)\,h(k)$$

or $$y(n) = x(n+2)h(-2) + x(n+1)h(-1) + x(n-1)h(1) + x(n-2)h(2)$$

$$= 4\sqrt{2}\,e^{j\frac{\pi}{4}(n+2)} - 2\sqrt{2}\,e^{j\frac{\pi}{4}(n+1)} - 2\sqrt{2}\,e^{j\frac{\pi}{4}(n-1)} + 4\sqrt{2}\,e^{j\frac{\pi}{4}(n-2)}$$

$$= 4\sqrt{2}\left[e^{j\frac{\pi}{4}(n+2)} + e^{j\frac{\pi}{4}(n-2)}\right] - 2\sqrt{2}\left[e^{j\frac{\pi}{4}(n+1)} + e^{j\frac{\pi}{4}(n-1)}\right]$$

$$= 4\sqrt{2}\,e^{j\frac{\pi}{4}n}\left[e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}}\right] - 2\sqrt{2}\,e^{j\frac{\pi}{4}n}\left[e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right]$$

$$= 4\sqrt{2}\,e^{j\frac{\pi}{4}n}[0] - 2\sqrt{2}\,e^{j\frac{\pi}{4}n}\left[2\cos\frac{\pi}{4}\right]$$

or $$y(n) = -4e^{j\frac{\pi}{4}n}$$

Sol. 82 Option (B) is correct.

From given graph the relation in $x(t)$ and $y(t)$ is

$$y(t) = -x[2(t+1)]$$

$$x(t) \xleftrightarrow{F} X(f)$$

Using scaling we have

$$x(at) \xleftrightarrow{F} \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

Thus $$x(2t) \xleftrightarrow{F} \frac{1}{2}X\left(\frac{f}{2}\right)$$

Using shifting property we get

$$x(t - t_0) = e^{-j2\pi f t_0}X(f)$$

Thus $$x[2(t+1)] \xleftrightarrow{F} e^{-j2\pi f(-1)}\frac{1}{2}X\left(\frac{f}{2}\right) = \frac{e^{j2\pi f}}{2}X\left(\frac{f}{2}\right)$$

$$-x[2(t+1)] \xleftrightarrow{F} -\frac{e^{j2\pi f}}{2}X\left(\frac{f}{2}\right)$$

Sol. 83 Option (C) is correct.

From the Final value theorem we have

$$\lim_{t \to \infty} i(t) = \lim_{s \to 0} sI(s) = \lim_{s \to 0} s\frac{2}{s(1+s)} = \lim_{s \to 0}\frac{2}{(1+s)} = 2$$

Sol. 84 Option (D) is correct.

Here $$C_3 = 3 + j5$$

For real periodic signal

$$C_{-k} = C_k^*$$

Thus $\quad C_{-3} = C_k = 3 - j5$

Sol. 85 Option (C) is correct.

$$y(t) = 4x(t-2)$$

Taking Fourier transform we get

$$Y(e^{j2pf}) = 4e^{-j2pf2} X(e^{j2pf}) \qquad \text{Time Shifting property}$$

or $$\frac{Y(e^{j2pf})}{X(e^{j2pf})} = 4e^{-4jpf}$$

Thus $$H(e^{j2pf}) = 4e^{-4jpf}$$

Sol. 86 Option (B) is correct.

We have $\quad h(n) = 3d(n-3)$

or $\quad H(z) = 2z^{-3} \qquad$ Taking z transform

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

Now $\quad Y(z) = H(z)X(z) = 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$

$$= 2(z + z^{-1} - 2z^{-2} + 2z^{-3} - 3z^{-7})$$

Taking inverse z transform we have

$$y(n) = 2[d(n+1) + d(n-1) - 2d(n-2)$$

$$+ 2d(n-3) - 3d(n-7)]$$

At $n = 4$, $\quad y(4) = 0$

Sol. 87 Option (A) is correct.

System is non causal because output depends on future value

For $n \# 1$ $\quad y(-1) = x(-1+1) = x(0)$

$$y(n - n_0) = x(n - n_0 + 1) \qquad \text{Time varying}$$

$$y(n) = x(n+1) \qquad \text{Depends on Future}$$

i.e. $$y(1) = x(2) \qquad \text{None causal}$$

For bounded input, system has bounded output. So it is stable.

$$y(n) = x(n) \text{ for } n \$ 1$$

$$= 0 \text{ for } n = 0$$

$$= x(x+1) \text{ for } n \# -1$$

So system is linear.

Sol. 88 Option (C) is correct.

The frequency response of RC-LPF is

$$H(f) = \frac{1}{1 + j2pfRC}$$

Now $\quad H(0) = 1$

$$\frac{|H(f_1)|}{H(0)} = \frac{1}{/1 + 4p^2 f_1^2 R^2 C^2} \$ 0.95$$

or $\quad 1 + 4p^2 f_1^2 R^2 C^2 \# 1.108$

or $\quad 4p^2 f_1^2 R^2 C^2 \# 0.108$

or $\quad 2pf_1 RC \# 0.329$

or $$f_1 \# \frac{0.329}{2pRC}$$

or $f_1 \# \dfrac{0.329}{2pRC}$

or $f_1 \# \dfrac{0.329}{2p1k \# 1m}$

or $f_1 \# 52.2 \text{ Hz}$

Thus $f_{1\,max} = 52.2\,\text{Hz}$

Sol. 89 Option (A) is correct.

$$H(w) = \frac{1}{1 + jwRC}$$

$$q(w) = -\tan^{-1} wRC$$

$$t_g = -\frac{dq(w)}{dw} = \frac{RC}{1 + w^2R^2C^2} = \frac{10^{-3}}{1 + 4p^2 \# 10^4 \# 10^{-6}} = 0.717 \text{ ms}$$

Sol. 90 Option (C) is correct.

If $x(t) * h(t) = g(t)$

Then $x(t - T_1) * h(t - T_2) = y(t - T_1 - T_2)$

Thus $x(t + 5) * d(t - 7) = x(t + 5 - 7) = x(t - 2)$

Sol. 91 Option (B) is correct.

In option (B) the given function is not periodic and does not satisfy Dirichlet condition. So it cant be expansion in Fourier series.

$$x(t) = 2\cos pt + 7\cos t$$

$$T_1 = \frac{2p}{w} = 2$$

$$T_2 = \frac{2p}{1} = 2p$$

$$\frac{T_1}{T_2} = \frac{1}{p} = \text{irrational}$$

Sol. 92 Option (C) is correct.

From the duality property of fourier transform we have

If $x(t) \xleftrightarrow{FT} X(f)$

Then $X(t) \xleftrightarrow{FT} x(-f)$

Therefore if $e^{-t}u(t) \xleftrightarrow{FT} \dfrac{1}{1 + j2pf}$

Then $\dfrac{1}{1 + j2pt} \xleftrightarrow{FT} e^{f}u(-f)$

Sol. 93 Option (A) is correct.

$$q(w) = -wt_0$$

$$t_p = \frac{-q(w)}{w} = t_0$$

and $t_g = -\dfrac{dq(w)}{dw} = t_0$

Thus $t_p = t_g = t_0 = \text{constant}$

Sol. 94 Option (*) is correct.

$$X(s) = \frac{5 - s}{s^2 - s - 2} = \frac{5 - s}{(s + 1)(s - 2)} = \frac{-2}{s + 1} + \frac{1}{s - 2}$$

Here three ROC may be possible.

$$\text{Re}(s) < -1$$
$$\text{Re}(s) > 2$$
$$-1 < \text{Re}(s) < 2$$

Since its Fourier transform exits, only $-1 < \text{Re}(s) < 2$ include imaginary axis. so this ROC is possible. For this ROC the inverse Laplace transform is

$$x(t) = [-2e^{-t}u(t) - 2e^{2t}u(-t)]$$

Sol. 95 Option (B) is correct.

For left sided sequence we have

$$-a^n u(-n-1) \xleftrightarrow{z} \frac{1}{1-az^{-1}} \qquad \text{where } |z| < a$$

Thus $$-5^n u(-n-1) \xleftrightarrow{z} \frac{1}{1-5z^{-1}} \qquad \text{where } |z| < 5$$

or $$-5^n u(-n-1) \xleftrightarrow{z} \frac{z}{z-5} \qquad \text{where } |z| < 5$$

Since ROC is $|z| < 5$ and it include unit circle, system is stable.

Alternative :

$$h(n) = -5^n u(-n-1)$$
$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=-\infty}^{-1} -5^n z^{-n} = -\sum_{n=-\infty}^{-1} (5z^{-1})^n$$

Let $n = -m$, then

$$H(z) = -\sum_{n=-1}^{-\infty} (5z^{-1})^{-m} = 1 - \sum_{m=0}^{\infty} (5^{-1}z)^{-m}$$
$$= 1 - \frac{1}{1-5^{-1}z}, \qquad |5^{-1}z| < 1 \text{ or } |z| < 5$$
$$= 1 - \frac{5}{5-z} = \frac{z}{z-5}$$

Sol. 96 Option (B) is correct.

$$\frac{1}{s^2(s-2)} = \frac{1}{s^2} \cdot \frac{1}{s-2}$$
$$\frac{1}{s^2} \cdot \frac{1}{s-2} \xleftrightarrow{L} (t * e^{2t}) u(t)$$

Here we have used property that convolution in time domain is multiplication in $s-$ domain

$$X_1(s) X_2(s) \xleftrightarrow{LT} x_1(t) * x_2(t)$$

Sol. 97 Option (A) is correct.

We have $h(n) = u(n)$

$$H(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n} = \sum_{n=0}^{\infty} 1 . z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n$$

$H(z)$ is convergent if

$$\sum_{n=0}^{\infty} (z^{-1})^n < \infty$$

and this is possible when $|z^{-1}| < 1$. Thus ROC is $|z^{-1}| < 1$ or $|z| > 1$

Sol. 98 Option (A) is correct.

We know that $\delta(t) x(t) = x(0) \delta(t)$ and $\int_{-\infty}^{\infty} \delta(t) = 1$

Let $x(t) = \cos(\frac{3}{2}t)$, then $x(0) = 1$

Now $$\int_{-\infty}^{\infty}\delta(t)x(t) = \int_{-\infty}^{\infty}x(0)\delta(t)\,dt = \int_{-\infty}^{\infty}\delta(t)\,dt = 1$$

Sol. 99 Option (B) is correct.

Let E be the energy of $f(t)$ and E_1 be the energy of $f(2t)$, then

$$E = \int_{-\infty}^{\infty}[f(t)]^2 dt$$

and $$E_1 = \int_{-\infty}^{\infty}[f(2t)]^2 dt$$

Substituting $2t = p$ we get
$$E_1 = \int_{-\infty}^{\infty}[f(p)]^2\frac{dp}{2} = \frac{1}{2}\int_{-\infty}^{\infty}[f(p)]^2 dp = \frac{E}{2}$$

Sol. 100 Option (B) is correct.

Since $h_1(t) \neq 0$ for $t < 0$, thus $h_1(t)$ is not causal

$h_2(t) = u(t)$ which is always time invariant, causal and stable.

$h_3(t) = \frac{u(t)}{1+t}$ is time variant.

$h_4(t) = e^{-3t}u(t)$ is time variant.

Sol. 101 Option (B) is correct.

$$h(t) = f(t) * g(t)$$

We know that convolution in time domain is multiplication in $s-$ domain.

$$f(t) * g(t) = h(t) \overset{L}{\longleftrightarrow} H(s) = F(s) \times G(s)$$

Thus $$H(s) = \frac{s+2}{s^2+1} \times \frac{s^2+1}{(s+2)(s+3)} = \frac{1}{s+3}$$

Sol. 102 Option (B) is correct.

Since normalized Gaussion function have Gaussion FT

Thus $$e^{-at^2} \overset{FT}{\longleftrightarrow} \sqrt{\frac{\pi}{a}}\, e^{-\frac{\pi^2 f^2}{a}}$$

Sol. 103 Option (B) is correct.

Let $$x(t) = ax_1(t) + bx_2(t)$$
$$ay_1(t) = atx_1(t)$$
$$by_2(t) = btx_2(t)$$

Adding above both equation we have

$$ay_1(t) + by_2(t) = atx_1(t) + btx_2(t) = t[ax_1(t) + bx_2(t)] = tx(t)$$

or $$ay_1(t) + by_2(t) = y(t)$$ Thus system is linear

If input is delayed then we have

$$y_d(d) = tx(t - t_0)$$

If output is delayed then we have

$$y(t - t_0) = (t - t_0)\,x(t - t_0)$$

which is not equal. Thus system is time varying.

Sol. 104 Option (A) is correct.

We have $$h(t) = e^{2t} \overset{LS}{\longleftrightarrow} H(s) = \frac{1}{s-2}$$

and $$x(t) = e^{3t} \overset{LS}{\longleftrightarrow} X(s) = \frac{1}{s-3}$$

Now output is $$Y(s) = H(s)\,X(s) = \frac{1}{s-2} \times \frac{1}{s-3} = \frac{1}{s-3} - \frac{1}{s-2}$$

Thus $$y(t) = e^{3t} - e^{2t}$$

Sol. 105 Option (C) is correct.

We know that for a square wave the Fourier series coefficient

$$C_{nsq} = \frac{AT}{T}\,\frac{\sin\frac{n\omega_0 T}{2}}{\frac{n\omega_0 T}{2}} \qquad \text{...(i)}$$

Thus $$C_{nsq} \propto \frac{1}{n}$$

If we integrate square wave, triangular wave will be obtained,

Hence $$C_{ntri} \propto \frac{1}{n^2}$$

Sol. 106 Option (B) is correct.

$$u(t) - u(t-1) = f(t) \xleftrightarrow{L} F(s) = \frac{1}{s}[1 - e^{-s}]$$

$$u(t) - u(t-2) = g(t) \xleftrightarrow{L} G(s) = \frac{1}{s}[1 - e^{-2s}]$$

$$f(t) * g(t) \xleftrightarrow{L} F(s)\,G(s)$$
$$= \frac{1}{s^2}[1 - e^{-s}][1 - e^{-2s}]$$
$$= \frac{1}{s^2}[1 - e^{-2s} - e^{-s} + e^{-3s}]$$

or $$f(t) * g(t) \xleftrightarrow{L} = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}$$

Taking inverse Laplace transform we have

$$f(t) * g(t) = t - (t-2)\,u(t-2) - (t-1)\,u(t-1) + (t-3)\,u(t-3)$$

The graph of option (B) satisfy this equation.

Sol. 107 Option (A) is correct.

Sol. 108 Option (A) is correct.

We have $$f(nT) = a^{nT}$$

Taking z -transform we get

$$F(z) = \sum_{n=-\infty}^{\infty} a^{nT} z^{-n} = \sum_{n=-\infty}^{\infty} (a^T)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{a^T}{z}\right)^n = \frac{z}{z - a^T}$$

Sol. 109 Option (B) is correct.

If $$\mathcal{L}[f(t)] = F(s)$$

Applying time shifting property we can write

$$\mathcal{L}[f(t-T)] = e^{-sT} F(s)$$

Sol. 110 Option (A) is correct.

Sol. 111 Option (A) is correct.

Sol. 112 Option (C) is correct.

Given z transform

$$C(z) = \frac{z^{-1}(1 - z^{-4})}{4(1 - z^{-1})^2}$$

Applying final value theorem

$$\lim_{n \to \infty} f(n) = \lim_{z \to 1} (z-1) f(z)$$

$$\lim_{z \to 1} (z-1)F(z) = \lim_{z \to 1} (z-1)\frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2} = \lim_{z \to 1} \frac{z^{-1}(1-z^{-4})(z-1)}{4(1-z^{-1})^2}$$

$$= \lim_{z \to 1} \frac{z^{-1}z^{-4}(z^4-1)(z-1)}{4z^{-2}(z-1)^2}$$

$$= \lim_{z \to 1} \frac{z^{-3}}{4} \frac{(z-1)(z+1)(z^2+1)(z-1)}{(z-1)^2}$$

$$= \lim_{z \to 1} \frac{z^{-3}}{4}(z+1)(z^2+1) = 1$$

Sol. 113 Option (A) is correct.

We have $F(s) = \frac{\omega}{s^2+\omega^2}$

$\lim_{t \to \infty} f(t)$ final value theorem states that:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

It must be noted that final value theorem can be applied only if poles lies in –ve half of s-plane.

Here poles are on imaginary axis $(s_1, s_2 = \pm j\omega)$ so can not apply final value theorem. so $\lim_{t \to \infty} f(t)$ cannot be determined.

Sol. 114 Option (D) is correct.

Trigonometric Fourier series of a function $x(t)$ is expressed as :

$$x(t) = A_0 + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

For even function $x(t)$, $B_n = 0$

So $$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos n\omega t$$

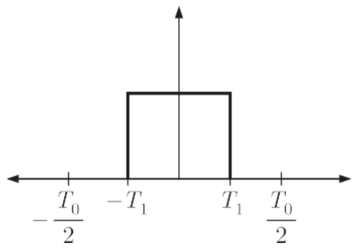
Series will contain only DC & cosine terms.

Sol. 115 Option (C) is correct.

Given periodic signal

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}$$

The figure is as shown below.



For $x(t)$ fourier series expression can be written as

$$x(t) = A_0 + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

where dc term

$$A_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt$$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{-T_1} x(t)\,dt + \int_{-T_1}^{T_1} x(t)\,dt + \int_{T_1}^{T_0/2} x(t)\,dt\right] = \frac{1}{T_0}[0 + 2T_1 + 0]$$

$$A_0 = \frac{2T_1}{T_0}$$

Sol. 116 Option (B) is correct.

The unit impulse response of a LTI system is $u(t)$

Let $h(t) = u(t)$

Taking LT we have $H(s) = \frac{1}{s}$

If the system excited with an input $x(t) = e^{-at}u(t)$, $a > 0$, the response

$$Y(s) = X(s)\,H(s)$$

$$X(s) = \mathcal{L}[x(t)] = \frac{1}{(s+a)}$$

so $$Y(s) = \frac{1}{(s+a)}\frac{1}{s} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right]$$

Taking inverse Laplace, the response will be

$$y(t) = \frac{1}{a}[1 - e^{-at}]$$

Sol. 117 Option (B) is correct.

We have $$x[n] = \sum_{k=0}^{\infty} \delta(n-k)$$

$$X(z) = \sum_{k=0}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty}\left[\sum_{k=0}^{\infty} \delta(n-k)z^{-n}\right]$$

Since $\delta(n-k)$ defined only for $n = k$ so

$$X(z) = \sum_{k=0}^{\infty} z^{-k} = \frac{1}{(1-1/z)} = \frac{z}{(z-1)}$$

Sol. 118 Option (B) is correct.

Sol. 119 Option (B) is correct.

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

by differentiation property;

$$\mathcal{F}\left[\frac{dx(t)}{dt}\right] = j\omega X(\omega)$$

or $$\mathcal{F}\left[\frac{dx(t)}{dt}\right] = j2\pi f X(f)$$

Sol. 120 Option (C) is correct.

We have $$f(t) \xleftrightarrow{\mathcal{F}} g(\omega)$$

by duality property of fourier transform we can write

$$g(t) \xleftrightarrow{\mathcal{F}} 2\pi f(-\omega)$$

so $$\mathcal{F}[g(t)] = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt = 2\pi f(-\omega)$$

Sol. 121 Option (B) is correct.

Given function

$$x(t) = e^{\alpha t}\cos(\alpha t)$$

Now $$\cos(\alpha t) \xleftrightarrow{\mathcal{L}} \frac{s}{s^2+\alpha^2}$$

If $$x(t) \xleftrightarrow{\mathcal{L}} X(s)$$

then $\quad e^{s_0 t} x(t) \overset{L}{\longleftrightarrow} X(s - s_0)$ $\quad$ shifting in s-domain

so $\quad e^{\alpha t}\cos(\alpha t) \overset{L}{\longleftrightarrow} \dfrac{(s-\alpha)}{(s-\alpha)^2 + \alpha^2}$

Sol. 122 Option (C) is correct.

For a function $x(t)$, trigonometric fourier series is :

$$x(t) = A_0 + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

where $\quad A_0 = \dfrac{1}{T_0}\int_{T_0} x(t)\,dt$ $\quad$ T_0 = Fundamental period

$$A_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega t\,dt$$

$$B_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega t\,dt$$

For an even function $x(t)$, coefficient $B_n = 0$

for an odd function $x(t)$, $\quad A_0 = 0$

$\quad A_n = 0$

so if $x(t)$ is even function its fourier series will not contain sine terms.

Sol. 123 Option (C) is correct.

The conjugation property allows us to show if $x(t)$ is real, then $X(j\omega)$ has conjugate symmetry, that is

$$X(-j\omega) = X^*(j\omega) \qquad [x(t)\ \text{real}]$$

Proof :

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

replace ω by $-\omega$ then

$$X(-j\omega) = \int_{-\infty}^{\infty} x(t)e^{j\omega t}\,dt$$

$$X^*(j\omega) = \left[\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt\right]^* = \int_{-\infty}^{\infty} x^*(t)e^{j\omega t}\,dt$$

if $x(t)$ real $x^*(t) = x(t)$

then $\quad X^*(j\omega) = \int_{-\infty}^{\infty} x(t)e^{j\omega t}\,dt = X(-j\omega)$

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